V.P & R.P.T.P.SCIENCE COLLEGE

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B.Sc.(MATHEMATICS) SEMESTER - 6

Multiple Choice Questions Of US06CMTH22

(Ring Theory)

By

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Unit-1

Que. Fill in the following blanks.

(1) .... is a non-commutative ring .
(a) $\mathbb{Z}$ (b) $\mathbb{Q}$ **(c)** $M_2(\mathbb{R})$ (d) none of these

(2) .... is a field .
(a) $\mathbb{Z}$ **(b)** $\mathbb{Q}$ (c) $M_2(\mathbb{R})$ (d) none of these

(3) .... is a Skew field but not a field .
(a) Ring of real quaternion (b) $\mathbb{Q}$ (c) $M_2(\mathbb{R})$ (d) $\mathbb{Z}$

(4) .... is a ring with zero divisor but not an integral domain .
(a) $\mathbb{Z}$ (b) $\mathbb{Q}$ **(c)** $M_2(\mathbb{R})$ (d) none of these

(5) .... is a ring with zero divisor but not an integral domain .
(a) $\mathbb{Z}$ (b) $\mathbb{Q}$ **(c)** Ring of real quaternion (d) none of these

(6) .... is a non-commutative ring with unit element .
(a) $\mathbb{Z}$ (b) $\mathbb{Q}$ **(c)** $M_2(\mathbb{R})$ (d) none of these

(7) .... is a non-commutative ring with unit element .
(a) $\mathbb{Z}$ (b) $\mathbb{Q}$ **(c)** Ring of real quaternion (d) none of these

(8) .... is regular element of $\mathbb{Z}_9$.
(a) 3 **(b)** 4 (c) 6 (d) none of these

(9) .... is regular element of $\{a + b\sqrt{-5}/a, b \in \mathbb{Z}\}$.
(a) 0 (b) $\{\pm i\}$ **(c)** $\{\pm 1\}$ (d) $\{1 + \sqrt{-5}\}$

(10) $\mathbb{Z}_p$ is a field if p is ....
(a) 4 (b) 6 (c) not prime **(d)** prime

(11) $\mathbb{Z}_p$ is not a field if p is ....
(a) 2 (b) 3 **(c)** not prime (d) prime

(12) Characteristic of every field is either zero or ....
(a) prime (b) 4 (c) not prime (d) integer

(13) .... is sabring of $\mathbb{Q}$.
(a) 0 **(b)** $\mathbb{Z}$ (c) $\{\pm 1\}$ (d) $\mathbb{N}$

(14) Let f be a ring homomorphism ,then prove that f is one-one iff Ker f = ...
(a) i (b) 1 (c) $\{\pm 1\}$ **(d)** $\{0\}$

(15) .... is regular element of $\mathbb{Z}_{20}$.
(a) 10 (b) 4 (c) 6 **(d)** none of these

(16) .... is regular element of $\mathbb{Z}_{20}$.
(a) 16 **(b)** 13 (c) 14 (d) 15

(17) .... is regular element of $\mathbb{Z}_{20}$.
(a) 4 (b) 5 (c) 6 **(d)** 7

(18) .... is not regular element of $\mathbb{Z}_{20}$.
(a) 18 (b) 19 (c) 17 (d) 9

(19) In Ring of real quaternion , $(1 - 2i - 3j - 2k)^{-1} =$ ....
(a) $\dfrac{1 - 2i - 3j - 2k}{18}$ **(b)** $\dfrac{1 + 2i + 3j + 2k}{18}$ (c) $\dfrac{-1 + 2i + 3j + 2k}{18}$ (d) $\dfrac{1 - 2i - 3j - 2k}{6}$

(20) In Ring of real quaternion , $(1 - 2i - 3j - 2k)i =$ ....
(a) $i + 2 - 3k - 2j$ (b) $i + 2 - 3k + 2j$ (c) $i - 2 - 3k - 2j$ **(d)** $i + 2 + 3k - 2j$

(21) In Ring of real quaternion , $(i - j)(i + j) =$ ....
(a) -2 (b) 1 **(c)** 0 (d) -1

(22) In ring $R = \{a + b\sqrt{-5} \ / \ a \ , \ b \in \mathbb{Q} \ \}$, $(-1 + 2\sqrt{-5})^{-1} =$ ....

(a) $\dfrac{1 - 2\sqrt{-5}}{21}$ (b) $\dfrac{-1 - 2\sqrt{-5}}{5}$ **(c)** $\dfrac{-1 - 2\sqrt{-5}}{21}$ (d) $\dfrac{-1 + 2\sqrt{-5}}{21}$

(23) Let R be the set of all subsets of a set X . Define $+$ *and* $\cdot$ in R by
$A + B = (A - B) \cup (B - A)$, $A \cdot B = A \cap B$, unit element of R is ....
(a) ϕ (b) 1 (c) R **(d)** X

(24) Let R be the set of all subsets of a set X . Define $+$ *and* $\cdot$ in R by
$A + B = (A - B) \cup (B - A)$, $A \cdot B = A \cap B$, additive identity of R is ....
(a) ϕ (b) 1 (c) R (d) X

(25) Let R be the set of all subsets of a set X . Define $+$ *and* $\cdot$ in R by
$A + B = (A - B) \cup (B - A)$, $A \cdot B = A \cap B$, then Ch R ....
(a) 1 **(b)** 2 (c) 0 (d) ϕ

(26) Cancellation laws are always satisfied in ....
(a) integral domain (b) ring (c) ring with unit element (d) commutative ring

(27) .... are regular elements of ring of Gaussian integer .
(a) $0, 1$ **(b)** $\pm i$ (c) ± 2 (d) $1 \pm i$

(28) .... are regular elements of ring of Gaussian integer .
(a) $0, i$ (b) $\pm 2i$ **(c)** ± 1 (d) $1 \pm i$

(29) Every integral domain can be imbedded in a ....
(a) $\mathbb{Z}$ (b) $\mathbb{N}$ **(c)** field (d) ring

(30) Quotient field of $\mathbb{Z}$ is ....
(a) $\mathbb{Z}$ **(b)** $\mathbb{Q}$ (c) $\mathbb{N}$ (d) Z_n

(31) Quotient field of ring of Gaussian integer is ....
(a) $\mathbb{Z}$ (b) $\mathbb{Q}$ (c) $\mathbb{Z} + i\,\mathbb{Z}$ **(d)** $\mathbb{Q} + i\,\mathbb{Q}$

(32) Quotient field of $2\mathbb{Z}$ is ....
(a) $\mathbb{Z}$ **(b)** $\mathbb{Q}$ (c) $2\mathbb{Q}$ (d) Z_n

(33) In ring $\mathbb{Z}$ .... is not invertible element .
(a) 1 (b) -1 **(c)** 2 (d) none of these

(34) In ring $\mathbb{Q}$ .... is not invertible element .
(a) 1 (b) -1 (c) 2 **(d)** 0

(35) In ring Z_n , $\bar{a} +$ .... $= \bar{0}$, $\forall$ $\bar{a} \in Z_n$.
(a) $-n$ **(b)** $\overline{n-a}$ (c) n (d) none of these

(36) $\mathbb{Z}_p$ is a field if p is ....
(a) 4 (b) 6 (c) 10 **(d)** 13

(37) $\mathbb{Z}_p$ is not a field if p is ....
(a) 2 (b) 3 **(c)** 8 (d) 13

(38) $\mathbb{Z}_p$ is an integral domain if p is ....
(a) 4 (b) 6 (c) 10 **(d)** 17

(39) $\mathbb{Z}_p$ is not an integral domain if p is ....
(a) 2 (b) 3 **(c)** 4 (d) 13

UNIT-2

(1) .... is an ideal in Z_6 .
(a) $\{\bar{0}, \bar{3}\}$ (b) $\{\bar{0}, \bar{2}\}$ (c) $\{\bar{0}, \bar{4}\}$ (d) $\{\bar{0}, \bar{5}\}$

(2) .... is an ideal in Z_6 .
(a) $\{\bar{4}, \bar{2}\}$ **(b)** $\{\bar{0}, \bar{2}, , \bar{4}\}$ (c) $\{\bar{0}, \bar{3}, \bar{4}\}$ (d) $\{\bar{0}, \bar{5}\}$

(3) .... is a simple ring .
(a) $\mathbb{Z}$ **(b)** $\mathbb{Q}$ (c) $\mathbb{N}$ (d) none of these

(4) .... is maximal ideal of field .
(a) 0 (b) $\{1\}$ **(c)** $\{0\}$ (d) none of these

(5) If R is ring then $R/\{0\} =$ ....
(a) 0 (b) $\{1\}$ (c) $\{0\}$ **(d)** R

(6) If $I = \{\bar{0}, \bar{2}, \bar{4}\}$ then $Z_6/I =$ ....
(a) $\{I, \bar{1} + I\}$ (b) $\{I\}$ (c) $\{\bar{1}\}$ (d) $\{I, \bar{2} + I\}$

(7) $Z/nZ =$ ....
(a) Z **(b)** Z_n (c) Z/Z_n (d) 1/n

(8) $Z/5Z =$ ....
(a) Z_5 (b) Z (c) Z_4 (d) 1/5

(9) If I is ideal in ring R and $a + I = I$ then ....
(a) $a = 0$ (b) $a = I$ **(c)** $a \in I$ (d) none of these

(10) If I is ideal in ring R and $a + I = b + I$ then ....
(a) $a + b \in I$ **(b)** $a - b \in I$ (c) $a = b$ (d) none of these

(11) If I is ideal in ring R then unit element of R/I is ....
(a) 0 (b) 1 (c) R **(d)** $1 + I$

(12) .... is an ideal in ring R .
(a) 0 (b) $\{1\}$ **(c)** $\{0\}$ (d) none of these

(13) $1 + 2i$ and .... are associates in the ring of Gaussian integer .
(a) $2 + i$ **(b)** $-2 + i$ (c) i (d) $2 + i$

(14) $1 + 2i$ and .... are associates in the ring of Gaussian integer .
(a) $2 + i$ (b) $-2 - i$ (c) i **(d)** $2 - i$

(15) If $n \in \mathbb{Z}$, $n > 1$ is irreducible then n is ....
(a) 4 (b) 0 **(c)** prime (d) not prime

(16) In $R = \{a + b\sqrt{-5}/a, b \in \mathbb{Z}\}$, $1 + 2\sqrt{-5}$ is .... in R .
(a) unit **(b)** irreducible (c) prime (d) non of these

(17) In $R = \{a + b\sqrt{-5}/a, b \in \mathbb{Z}\}$, $1 + 2\sqrt{-5}$ is .... in R .
(a) unit (b) not irreducible (c) prime **(d)** not prime

(18) In $R = \{a + b\sqrt{-5}/a, b \in \mathbb{Z}\}$, $1 + 2\sqrt{-5}$ is .... in R .
(a) not unit (b) not irreducible (c) prime (d) unit

(19) In $R = \{a + b\sqrt{-5}/a, b \in \mathbb{Z}\}$, gcd of $1 + 2\sqrt{-5}$ and 3 is ....
(a) unit (b) not exist (c) prime (d) not unit

(20) In $R = \{a + b\sqrt{-5}/a, b \in \mathbb{Z}\}$, gcd of $2 + 2\sqrt{-5}$ and 6 is ....
(a) unit **(b)** not exist (c) prime (d) not unit

(21) Cancellation laws are always satisfied in ....
(a) integral domain (b) ring (c) ring with unit element (d) commutative ring

(22) In $\mathbb{Z} + i\mathbb{Z}$, gcd of 2 and $-1 + 5i$ is ....
(a) $2 + i$ (b) $2 - i$ (c) i **(d)** $1 - i$

(23) In $\mathbb{Z} + i\mathbb{Z}$, gcd of $1 + i$ and $-1 + 5i$ is ....
(a) 1 (b) $-1 + 5i$ **(c)** $1 + i$ (d) $1 - i$

(24) If R is commutative ring with 1 and $Ra \subset Rb$ then ....
(a) a = b (b) $a \subset b$ (c) a/b **(d)** b/a

(25) If R is commutative ring with 1 and b/a then ....
(a) a = b **(b)** $Ra \subset Rb$ (c) $Rb \subset Ra$ (d) $Ra = Rb$

(26) Let R be an integral domain and $a \in R$ is an irreducible element then a is ....
(a) unit (b) not exist (c) prime (d) not unit

(27) In $R = \{a + b\sqrt{-5}/a, b \in \mathbb{Z}\}$,...................... is irreducible in R .
(a) $3 + \sqrt{-5}$ (b) 1 (c) $8 + 6\sqrt{-5}$ (d) $6 + 2\sqrt{-5}$

(28) In $R = \{a + b\sqrt{-5}/a, b \in \mathbb{Z}\}$,...................... is irreducible in R .
(a) -1 **(b)** $4 + 3\sqrt{-5}$ (c) $8 + 6\sqrt{-5}$ (d) $6 + 2\sqrt{-5}$

(29) .... is not a field .
(a) $\mathbb{Z}/2\mathbb{Z}$ **(b)** $\mathbb{Z}/4\mathbb{Z}$ (c) $\mathbb{Z}/11\mathbb{Z}$ (d) $\mathbb{Z}/5\mathbb{Z}$

(30) .... is a field .
(a) $\mathbb{Z}/6\mathbb{Z}$ (b) $\mathbb{Z}/4\mathbb{Z}$ (c) $\mathbb{Z}/12\mathbb{Z}$ **(d)** $\mathbb{Z}/5\mathbb{Z}$

(31) Characteristic of $\mathbb{Z}_2 \times \mathbb{Z}_4 \times \mathbb{Z}_6$ is ....
(a) 2 (b) 4 (c) 6 **(d)** 12

(32) The number of prime ideals of $\mathbb{Z}_{10^5}$ is ....
(a) 2 (b) 10 (c) 5 (d) 10^5

UNIT-3

(1) Every .... is Principal ideal domain .
(a) integral domain (b) ring **(c)** Euclidean domain (d) commutative ring

(2) Every .... has unit element.
(a) integral domain (b) ring **(c)** Euclidean domain (d) commutative ring

(3) Let R be Euclidean domain , $a, b \in R$, a is proper divisor of b then $d(b)$..................$d(a)$.
(a) $=$ (b) $\leq$ **(c)** $>$ (d) $<$

(4) Let R be Euclidean domain , $a, b \in R$, a is proper divisor of b then $d(a)$..................$d(b)$.
(a) $=$ (b) $\subset$ (c) $>$ **(d)** $<$

(5) Let R be Euclidean domain , $a \in R$ is unit , then $d(a) =$ ....
(a) 0 **(b)** $d(1)$ (c) $d(2)$ (d) 1

(6) In ring of Gaussian integer , $2 - i =$ ....$(1 + 2i)$
(a) i (b) 2 (c) $1 + i$ **(d)** $-i$

(7) In ring of Gaussian integer , $1 + 2i =$ ....$(2 - i)$
(a) i (b) 2 (c) $1 + i$ (d) $-i$

(8) .... is Euclidean domain .
(a) $\{0, 1\}$ (b) $\{0\}$ (c) $\mathbb{N}$ **(d)** $\mathbb{Z}$

(9) .... is Euclidean domain .
(a) $\{0, 1\}$ (b) $\{0\}$ (c) $\mathbb{N} + i\mathbb{N}$ **(d)** $\mathbb{Z} + i\mathbb{Z}$

(10) .... is Factorization domain .
(a) $\{0,1\}$ (b) $\{0\}$ (c) $\mathbb{N}$ **(d)** $\mathbb{Z}$

(11) .... is Factorization domain .
(a) $\{0,1\}$ (b) $\{0\}$ (c) $\mathbb{N}+i\mathbb{N}$ **(d)** $\mathbb{Z}+i\mathbb{Z}$

(12) .... is Principal ideal domain .
(a) $\{0,1\}$ (b) $\{0\}$ (c) $\mathbb{N}$ **(d)** $\mathbb{Z}$

(13) .... is Principal ideal domain .
(a) $\{0,1\}$ (b) $\{0\}$ (c) $\mathbb{N}+i\mathbb{N}$ **(d)** $\mathbb{Z}+i\mathbb{Z}$

(14) Every irreducible element in unique factorization domain is ....
(a) unit (b) not unit **(c)** prime (d) not unit

(15) If every irreducible element is.................. in factorization domain R then R is unique factorization domain .
(a) unit (b) not unit **(c)** prime (d) not unit

(16) Every .... is unique factorization domain .
(a) integral domain (b) ring **(c)** Euclidean domain (d) commutative ring

(17) Every .... is unique factorization domain .
(a) integral domain (b) ring **(c)** principle ideal domain (d) commutative ring

UNIT-4

(1) If R is commutative ring , $f(x), g(x) \in R[x]$ then degree(fg) .... degree f + degree g .
(a) $>$ **(b)** $\leq$ (c) $=$ (d) $\geq$

(2) If R is an integral domain , $f(x), g(x) \in R[x]$ then degree(fg) .... degree f + degree g.
(a) $>$ (b) $\leq$ **(c)** $=$ (d) $\geq$

(3) If R is field , $f(x), g(x) \in R[x]$ then degree(fg) .... degree f + degree g .
(a) $>$ (b) $\leq$ **(c)** $=$ (d) $\geq$

(4) If F is field , $f(x) \in F[x]$, $\alpha \in F$ is a root of $f(x)$ then ....
(a) $(x-\alpha)/f(x)$ (b) $(x+\alpha)/f(x)$ (c) $f(x)/(x-\alpha)$ (d) $f(x)/(x+\alpha)$

(5) If R is integral domain , $f(x) \in R[x]$, degree of f is n then $f(x)$ has .... distinct roots in R .
(a) 2 (b) atleast n **(c)** atmost n (d) n

(6) If F is field, $f(x) \in F[x]$, degree of f is n then $f(x)$ has ....distinct roots in F .
(a) 2 (b) atleast n **(c)** atmost n (d) n

(7) If $R = Z + iZ$, $f(x) = 2x^2 - (1+i)x - 2$ then content of f is ....
(a) $2+i$ (b) $2-i$ (c) $1-i$ **(d)** $1+i$

(8) If R is an integral domain then $R[x]$ is ....
(a) integral domain (b) ring (c) Euclidean domain (d) field

(9) If F is field then $F[x]$ is ....
(a) $\{0\}$ (b) F **(c)** Euclidean domain (d) field

(10) If F is field then $F[x]$ is ....
(a) $\{0\}$ (b) F **(c)** principle ideal domain (d) field

(11) If F is field then $F[x]$ is ....
(a) $\{0\}$ (b) F **(c)** unique factorization domain (d) field

(12) Let $R = \mathbb{Z} + i\mathbb{Z}$, $f(x) = 2x^2 - (1-i)x - 2 \in R[x]$ then $C(f) =$ ....
(a) 1 (b) $1+i$ **(c)** $1-i$ (d) i

(13) Let $f(x) = 3x^3 - 2x^2 + 6x + 9 \in \mathbb{Z}[x]$ then $C(f) =$ ....
(a) 1 (b) -1 (c) i (d) $-i$

(14) $f(x) = x^2 + 8x - 2$ is irreducible over ....
(a) $\mathbb{Q}$ (b) $\mathbb{Z}$ (c) $\mathbb{N}$ (d) none of these

(15) .... is irreducible over $\mathbb{Z}$
(a) $x^2 - 5x + 6$ (b) $x^2 - 7x + 12$ (c) $x^2 - 9x + 20$ **(d)** none of these

(16) Degree of $\mathbb{Q}(\sqrt{3}\,,\ \sqrt{5})$ over $\mathbb{Q}$ is ....
(a) 3 (b) 1/2 **(c)** 4 (d) 1

(17) Degree of $\mathbb{Q}(\sqrt[3]{3}\,,\ \sqrt{2})$ over $\mathbb{Q}(\sqrt[3]{3})$ is ....
(a) 3 **(b)** 2 (c) $i/2$ (d) 1/3

(18) Degree of $\mathbb{Q}(\sqrt[3]{5}\,,\ \sqrt{5})$ over $\mathbb{Q}(\sqrt{5})$ is ....
(a) 3 (b) 2 (c) $i/2$ (d) 6

(19) Degree of $\mathbb{Q}(\sqrt[3]{7}\,,\ \sqrt{3})$ over $\mathbb{Q}$ is ....
(a) 3 (b) 2 (c) $i/2$ **(d)** 6

(20) .... is extension of $\mathbb{Q}$.
(a) $\mathbb{N}$ (b) $\mathbb{Z}$ (c) $\mathbb{Q}$ **(d)** $\mathbb{R}$

(21) .... is extension of $\mathbb{Q}$.
(a) $\mathbb{N}$ (b) $\mathbb{Z}$ (c) $\mathbb{Q}$ **(d)** $\mathbb{C}$

(22) .... is extension of $\mathbb{R}$.
(a) $\mathbb{N}$ (b) $\mathbb{Z}$ (c) $\mathbb{Q}$ **(d)** $\mathbb{R}$

(23) K/F is said to be simple extension if ....
(a) $F = K(\alpha)$ **(b)** $K = F(\alpha)$ (c) $K = F$ (d) None of these

(24) i is .... over $\mathbb{R}$.
(a) algebraic (b) transcendental (c) simple (d) extension

(25) e is .... over $\mathbb{R}$.
(a) algebraic **(b)** transcendental (c) simple (d) extension

(26) π is .... over $\mathbb{R}$.
(a) algebraic **(b)** transcendental (c) simple (d) extension

(27) $\sqrt[3]{2}$ is .... over $\mathbb{Q}$.
(a) algebraic (b) transcendental (c) simple (d) extension

(28) If α is algebraic over F with minimum polynomial of degree 2 then ....
(a) $[F(\alpha : F)] = 1$ (b) $[F(\alpha : F)] = 4$ **(c)** $[F(\alpha : F)] = 2$ (d) none of these

(29) If K/F is finite extension and $\alpha \in K$ with minimum polynomial of degree n then ....
(a) $n = [K : F]$ **(b)** $n/[K : F]$ (c) $[K : F]/n$ (d) None of these

(30) $\bar{\mathbb{Q}}/\mathbb{Q}$ is .... extension .
(a) algebraic (b) transcendental (c) not algebraic (d) finite

(31) $\bar{\mathbb{Q}}/\mathbb{Q}$ is .... extension .
(a) transcendental **(b)** not finite (c) not algebraic (d) finite

(32) .... is algebraically closed field .
(a) $\mathbb{Q}$ (b) $\mathbb{R}$ **(c)** $\bar{\mathbb{Q}}$ (d) $\bar{\mathbb{C}}$

(33) .... is algebraically closed field .
(a) $\mathbb{Q}$ (b) $\mathbb{R}$ (c) $\bar{\mathbb{C}}$ **(d)** $\mathbb{C}$

(34) .... is algebraic closure of $\mathbb{Q}$.
(a) $\mathbb{Q}$ (b) $\mathbb{R}$ **(c)** $\bar{\mathbb{Q}}$ (d) $\mathbb{C}$

(35) .... is algebraic closure of $\mathbb{R}$.
(a) $\mathbb{Q}$ **(b)** $\mathbb{C}$ (c) $\bar{\mathbb{Q}}$ (d) $\mathbb{R}$

(36) Degree of $\mathbb{Q}(\sqrt{3}\ ,\ \sqrt{5}\ ,\ \sqrt{2})/\mathbb{Q}$ is ....
(a) 2 (b) 6 **(c)** 8 (d) 4

(37) Degree of $\mathbb{Q}(\sqrt[5]{2}\ ,\ \sqrt{3}\ )/\mathbb{Q}$ is ....
(a) 10 (b) 6 (c) 15 (d) 5