

SEAT No. _____

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[70]

Sardar Patel University, Vallabh Vidyanagar

B.Sc Sem: V Subject : Mathematics

Date: 24/12/2020 US05CMTH21 [Real Analysis] Time: 2.00 to 4.00 Max.Marks: 70

Q.1 Choose the correct option for each of the following.

[10]

(1) Which of the following is an ordered field ?

(a) \mathbb{Q} (b) \mathbb{R} (c) \mathbb{N} (d) none of these

(2) The Smallest number of a set, if exists is

(a) the supremum of the set (b) the infimum of the set (c) not unique (d) none

(3) The supremum of $\left\{\frac{1}{m} + \frac{1}{n} / m, n \in \mathbb{N}\right\}$ is

(a) 0 (b) 1 (c) 2 (d) none

(4) Every open interval in \mathbb{R} isset

(a) an open (b) a closed (c) open and closed (d) none

(5) The derived set of $A = \{1, 2, 3, 4\}$ is

(a) A (b) \mathbb{R} (c) \emptyset (d) \mathbb{Z}

(6) The closure of \mathbb{Q} i.e $\bar{\mathbb{Q}}$ is

(a) \mathbb{N} (b) \mathbb{Q} (c) \emptyset (d) \mathbb{R}

(7) The Range of sequence is always

(a) empty (b) infinite (c) non- empty (d) none

(8) Every convergent sequence is

(a) oscillating (b) bounded (c) unbounded (d) none

(9) A positive term series $\sum \frac{1}{n^p}$ is convergent iff _____.

(a) $p = 1$ (b) $0 < p < 1$ (c) $p > 1$ (d) $p < 0$

(10) A series $\sum u_n$ is convergent then $\lim_{n \rightarrow \infty} u_n$

(a) $\neq 0$ (b) $= 0$ (c) $= 1$ (d) does not exists



[1]

(P.T.O.)



Q.2 Do as directed.

[8]

(1) The infimum of $\left\{\frac{(-1)^n}{n} / n \in \mathbb{N}\right\}$ is

(2) If $S = (0,5) \cup \{5,6,7\}$ then the greatest element of S is

(3) If $S_n = \left(\frac{-1}{n}, \frac{1}{n}\right), \forall n \in \mathbb{N}$ then $\bigcap_{n=1}^{\infty} S_n = \dots\dots\dots$

(4) is a limit point of a set $S = \left\{\frac{1}{n} / n \in \mathbb{N}\right\}$

(5) The range of sequence $\{(-1)^n / n \in \mathbb{N}\}$ is

(6) True Or False: A sequence without limit point is bounded.

(7) True Or False :if a positive term series $\sum_{n=1}^{\infty} u_n$ is convergent then its partial sums is bounded above.

(8) True Or False :The series $\sum_{n=1}^{\infty} \frac{n}{n+1}$ is convergent.

Q.3 Attempt anyTEN:

[20]

(1) Define : An Ordered Field.

(2) Prove that the greatest lower bound of a set S is unique ,if it exist.

(3) Find the g.l.b and l.u.b of $\left\{1 + \frac{(-1)^n}{n} / n \in \mathbb{N}\right\}$ if they exist.

(4) Prove that every open set is a union of open intervals.

(5) Define: An open set.

(6) Define: A limit point of a set .

(7) Define: A convergent sequence .

(8) Prove that every convergent sequence is bounded.

(9) Prove that $\lim_{n \rightarrow \infty} \frac{3+\sqrt{2}n}{n} = \sqrt{2}$

(10) Define : Infinite series

(11) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n!}$ is convergent.

(12) Investigate the behaviour of the series whose n^{th} term is $n \frac{1}{n}$.

[23]



Q.4 Attempt any FOUR :

[32]

- (1) State and Prove the Archimedean property of \mathbb{R} .
- (2) Prove that the set of all rationals \mathbb{Q} is not an order complete field.
- (3) Prove that a set is closed iff its complement is open.
- (4) Prove that the union of arbitrary family of open sets is open.
- (5) State and Prove Bolzano-Weierstrass theorem for sequence.
- (6) State and Prove Cauchy's first theorem on limits.
- (7) State and Prove comparison test of first type.
- (8) Prove that the positive term geometric series $1 + r + r^2 + \dots$ converges for $r < 1$ and diverges to ∞ for $r \geq 1$.

————— X —————

[3]