SEAT No.________

No. of Printed Pages : 03

[57]

SADAR PATEL UNIVERSITY

Sixth Semester B. Sc. Examination – 2021

Thurday, 15th July,-2021

Time: 10:00am to 12:00 pm

PHYSICS: US06CPHY21(Quantum Mechanics)

Total Marks: 70

Note: All the symbols have their usual meaning.

Que-1 Choose correct option to answer the question. [10]

(1) Potential energy of bound particle is ______.

(a) positive (b) zero (c) negative (d) infinite.

(2) Operator form of momentum in three dimension is taken as ______

(a) $-i\hbar\vec{\nabla}$ (b) $i\hbar\frac{\partial}{\partial t}$ (c) $i\hbar\int\frac{\partial}{\partial t}$ (d) $i\hbar\frac{\partial^2}{\partial t^2}$

(3) For a free state (zero potential) energy value of a particle is ______.

(a) discrete (b) continuous (c) always zero (d) infinite

(4) Expectation value of a self adjoint operator is ______.

(a) real (b) infinite (c) always 0 (d) imaginary.

(5) For any operator A and a wave function ϕ_a if $A\phi_a = a\phi_a$ then a is called ______.

(a) eigen function (b) probability amplitude
(c) probability density (d) eigen value

(6) If A is an operator and $A^\dagger$ is an adjoint operator of A then $(A^\dagger)^\dagger =$ ______.

(a) $A^\dagger$ (b) A^* (c) A (d) 1

(7) If Operators A and B are canonically conjugate operators then $[A,B] =$ ______.

(a) $i\hbar$ (b) $\hbar$ (c) $\frac{1}{2}i\hbar$ (d) $\frac{1}{3}i\hbar$

(8) For simple harmonic oscillator potential energy in one dimension is given by ______.

(a) $m\omega$ (b) $\frac{1}{2}Kx^2$ (c) $\frac{p^2}{2m}$ (d) mgh

(9) Angular momentum is defined as $L =$ ______.

(a) $\vec{r}\times\vec{p}$ (b) $\vec{r}\times\vec{p}^2$ (c) $\vec{r}\cdot\vec{p}$ (d) mv

(10) In a rigid body distance between two particles is ______.

(a) variable (b) zero (c) infinite (d) constant

(P.T.O.)

Que-2 **Fill the Blanks or State True or False as Required.** **[08]**

(1) For a square well of width 2a if $\Delta = \frac{h^2}{2ma^2}$ then Δ has the unit of ____.

(2) Expectation value of an operator A in quantum mechanics is given by $< A > = \int \psi^* \psi A d\tau$ [State True or False]

(3) If $\boldsymbol{\delta_{m,n}}$ is Kronecker delta function then $\boldsymbol{\delta_{m,n} = 0}$ when _____.

(4) If A and B are non-commutative self adjoint operators then $(AB)^\dagger = AB$ [State True or False]

(5) For canonically conjugate pair of operator A and B $(\Delta A)(\Delta B) \geq$ _________

(6) For two system of non-interacting particle Hamiltonian $H(1,2) = H_1(1) + H_2(2)$ [State True or False]

(7) Energy eigen value of an isotropic oscillator is given by E = _______.

(8) Central potential is a function of direction only. [State True or False]

Que-3 Answer briefly <u>any ten</u> of the following questions. **[20]**

(1) Discuss briefly penetration of particle in a classical forbidden region.

(2) For a square well potential draw diagrams showing wave functions of odd parity with proper notations.

(3) Give the interpretation of the quantity $\Delta = \frac{h^2}{2ma^2}$ appearing in the discussion of square well potential. Also show that the quantity $\frac{\Delta}{V}$ is dimension less.

(4) Explain adjoint operator. Also define self adjoint operator.

(5) Define non-degenerate and degenerate eigen values.

(6) What is observable? Also state expansion postulate.

(7) Write down fourth postulate of quantum mechanics.

(8) Define non-interacting particles.

(9) Write down any one property of identical particle.

(10) Write down expression for ∇^2 in spherical polar coordinates.

(11) What is rigid rotator? State the expression for its energy level separation. What is importance of studying rigid rotator?

(12) What is isotropic oscillator? Write down expressions for its energy.

Que-4 Long Answer Questions [Attempt any 4 out of 8] (Each question carries equal marks) **[32]**

(1) Draw a figure of square well potential and write down equations for potentials of its different regions. For bound states in square well potential obtain admissible solutions of wave functions.

(2) Using the admissibility solutions of a square well potential graphically show that in a square well potential energy levels are finite and discrete. Also briefly discuss parity of eigen functions.

(3) Define an adjoint operator and a self adjoint operator. Show that any two eigen functions belonging to distinct (unequal) eigen values of a self adjoint operator are mutually orthogonal.

(4) State expansion postulate. Show that the eigen functions belonging to discrete eigen values are normalizable and eigen functions belonging to continuous eigen values are of infinite norm

(5) For quantum mechanical observables A and B obtain the following expression;

$$(\Delta\mathcal{A})^2(\Delta\mathcal{B})^2 \geq -\frac{1}{4}\langle[A, B]\rangle^2$$

Where

$\mathcal{A} = A - \langle A\rangle$ and $\mathcal{B} = B - \langle B\rangle$

showing uncertainty in their measurements. Also show that if A & B are canonically conjugate pair of operators then,

$$(\Delta A)(\Delta B) \geq \frac{1}{2}\hbar$$

(6) For a simple harmonic oscillator the Hamiltonian is given by,

$$H = \frac{P^2}{2m} + \frac{1}{2}kx^2$$

In this case obtain Schrödinger equation as;

$$\frac{d^2u}{d\rho^2} + [\lambda - \rho^2]u = 0$$

Also obtain an expression for its energy eigen value as;

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c$$

(7) Obtain operator form of L^2 in terms of spherical polar coordinates

$$L^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

(8) Write down expression for Hamiltonian of anisotropic oscillator in three dimension and obtain the equation;

$$\nabla^2 u + \frac{2m}{\hbar^2}[E - V(r)]u = 0$$

Also define rigid rotator and show that its energy levels are not equispaced..

—x—