SEAT No.________ No. of Printed Pages : 03

SARDAR PATEL UNIVERSITY (B. Sc. Sem.5 Examination)

[145] **MATHEMTICS - US05CMTH22**

THEORY OF REAL FUNCTIONS

24th November 2021, Wednesday

Time:03:00 to 05:00 p.m. **Total Marks: 70**

Note: Figures to the right indicates the full marks.

Q:1 Answer thefollowing by selecting the correct choice from the given options. **[10]**

1. f(x) = |x| is ______at x = 0
   (a) discontinuous (b) differentiable
   (c) not differentiable (d) none
2. $\left(\frac{1}{f}\right)'(c) =$ ________
   (a) $\frac{f(c)}{\{f'(c)\}^2}$ (b) $\frac{-f(c)}{\{f'(c)\}^2}$ (c) $\frac{f'(c)}{\{f(c)\}^2}$ (d) $\frac{-f'(c)}{\{f(c)\}^2}$
3. If $f(x_1) \le f(x_2)$, $\forall x_1 \le x_2$ then the function f is said to be ________
   (a)decreasing (b) increasing
   (c) strictly decreasing (d) strictly increasing
4. Taylor's Remainder after n term is ______
   (a) $\frac{h^n(1-\theta)^{n-p}}{p(n-1)!} f^{(n)}(a+\theta h)$ (b) $\frac{h^p(1-\theta)^{n-p}}{p(n-1)!} f^{(n)}(a+\theta h)$
   (c) $\frac{h^n(1-\theta)^{n-p}}{(p-1)n!} f^{(n)}(a+\theta h)$ (d) $\frac{h^p(1-\theta)^{n-p}}{(p-1)n!} f^{(n)}(a+\theta h)$
5. $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots =$ ________
   (a)e^x (b) $\log(1+x)$ (c)$\sin x$ (d) $\cos x$
6. ________is an implicit function
   (a) $x^m = y^n$ (b) $x^2 = y^2$
   (c)$x^y = y^x$ (d) $x + 2xy + y^2 = 0$
7. The condition in Young's theorem is ________
   (a)necessary (b) sufficient
   (c)necessary and sufficient (d)not valid
8. $(\frac{\partial}{\partial x}dx + \frac{\partial}{\partial y}dy)^n\, z =$ ________
   (a) $\frac{\partial^n z}{\partial x^n} + \frac{\partial^n z}{\partial y^n}$ (b) $\frac{d^n z}{dx^n} + \frac{d^n z}{dy^n}$ (c) $\partial^n z$ (d) $d^n z$

9. For a sufficiently many times differentiable function $f(x, y)$ it's Taylor's expansion about (1,-2) is a series in powers of
(a) $x + 1 \& y - 2$ (b) $x - 1 \& y + 2$
(c) $x - 1 \& y - 2$ (d) $x + 1 \& y + 2$

10. $f(a, b)$ is an extreme value of f if ________
(a) $AC - B^2 \neq 0$ (b) $AC - B^2 < 0$
(c) $AC - B^2 = 0$ (d) $AC - B^2 > 0$

Q:2 Answer the given statement is TRUE or FALSE **[08]**

1. $f(x) = x - [x]$ is continuous at $x = 1$
2. A continuous function on a closed interval is also uniform continuous on that interval
3. $f(x) = \tan^{-1} x$ is strictly increasing
4. For the function $f(x) = e^x$ on [0,1] the value of c in mean value theorem will be 1
5. Sufficient condition for equality of f_{xy} and f_{yx} at a point is f_x and f_y are both differentiable at that point
6. Taylor's theorem is mainly used in expressing the function as sum of infinite terms
7. For $f(x, y) = x^3 - 3xe^y$, the value of $f_x(0,1)$ is 2
8. $AC - B^2 < 0$ indicates that fhas maxima

Q:3 Answer ANY TEN of the following. **[20]**

1. Evaluate $\lim_{x \to 1} \frac{|x|}{x}$
2. Show that a function which is derivable at a point is necessarily continuous at that point.
3. Examine continuity of $f(x) = x - [x]$ at $x = 3$
4. Define: increasing function
5. State Darboux theorem
6. State Rolle's theorem
7. Find repeated limits of $f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$ at (0,0)
8. Find simultaneous limit of $f(x, y) = \frac{2xy^2}{x^2 + y^4}$ at (0,0) if it exists
9. Using definition of partial derivatives find f_x and f_y of

$$f(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2} & if\ (x, y) \neq (0,0) \\ 0 & if\ (x, y) = (0,0) \end{cases}$$

10. State Maclaurin's theorem

11. Expand $e^x \tan^{-1} y$ up to first degree powers of $(x-1)$ and $(y-1)$

12. State necessary condition for a function f to have local extremum (a, b).

Q:4 Answer ANY FOUR of the following. [32]

(1) A function f is defined on R by

$$f(x) = \begin{cases} -x^2; & x \le 0 \\ 5x-4 & ;\ 0 < x \le 1 \\ 4x^2 - 3x & ;\ 1 < x < 2 \\ 3x+4\ ; & x \ge 2 \end{cases}$$

Examine continuity of f at $x = 0,1,2$. Also discuss the kind of discontinuity if any

(2) If f and g be two functions defined on some neighbourhood such that $\lim_{x\to c} f(x) = l$, $\lim_{x\to c} g(x) = m$ then prove that

$$\lim_{x\to c} f(x)\cdot g(x) = l\cdot m$$

(3) State and prove Lagrange's Mean Value theorem. Also discuss its geometric interpretation.

(4) Show that $\log(1+x)$ lies between $x - \frac{x^2}{2}$ and $x - \frac{x^2}{2(1+x)}$,

$$\forall x > 0$$

(5) Using definition of limit prove that $\lim_{(x,y)\to(0,0)} xy\frac{x^2-y^2}{x^2+y^2} = 0$

(6) Using definition of continuity show that the function

$f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & if\ (x,y) \ne (0,0) \\ 0 & if\ (x,y) = (0,0) \end{cases}$ is continuous at origin.

(7) Expand $x^2y + 3y - 2$ in powers of $(x-1)$&$(y+2)$

(8) Find maxima and minima of $x^3 + y^3 - 3x - 12y + 20$

—x—