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SARDAR PATEL UNIVERSITY

BSc Sem V Examination, Mathematics

US05CMTH23-Group Theory

Date : 25/11/21 **Time : 3-00 TO 5-00 P.m.**

Q.1 Choose the correct options. **(10)**

1. The order of -1 in the multiplicative group of non zero rational numbers is______.

   a. infinite b. 1 c. 2 d. none

2. The order of the group S_5 is _____

   a.4 b. 5 c. $4!$ d. $5!$

3. In Klein 4-group $= \{e, a, b, c\}$, $a^2 =$ _______.

   a.e b.b c. a d. c

4. The Generators of cyclic group $G = \{All\ fourth\ roots\ of\ unity\}$ under multiplication is _____ .

   a.$1, -1$ b. i, i^3 c. ω, ω^2 d. none

5. If $H = 7\mathbb{Z}$ is a subgroup of additive group $G = \mathbb{Z}$ then the index $(G : H) =$ _____

   a.3 b. 5 c. 2 d. 7

6. The cyclic group of order 7 has only _______generator.

   a.7 b. 6 c. 4 d. 1

7. If $\emptyset$ is Euler's function then $\emptyset(12) =$ ______.

   a.3 b. 11 c. 2 d. 4

8. S_n is group______.

   a. Klein 4-group b. cyclic c. commutative d. non-commutative

9. A permutation σ is an odd permutation if signature of σ is ______.

   a.1 b. -1 c. 2 d. none

10. The external direct sum of Z_2 is _____.

    a.$\mathbb{Q}$ b. $\mathbb{Z}$ c. $\mathbb{Z}_2$ d. Klein 4-group

Q.2 Do as directed. **(8)**

1) The group $(G, \cdot)$ of all 2×2 non-singular matrices is commutative group(True/False).
2) The multiplicative inverse of 6 in Z_7^* is ____(6/7).
3) Every group has at least one subgroup (True/False).
4) Fill in the blank: (1 2 3 4 5) is a ________ permutation (even/odd).

5) Every isomorphism is homomorphism (True/False).
6) Fill in the blank: $(b^{-1}a^{-1})^{-1} =$ ______.
7) An index of a subgroup is the number of elements in it (True/False).
8) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ is an identity permutation (True/False).

Q.3 Answer any TEN. **(20)**

1) Prove that every element of group G has unique inverse.
2) Does the set $(Z_8^*, \cdot)$ form a group? Justify.
3) Prove that every cyclic group is abelian.
4) If an order of a group is 10 then what are possible orders of it's subgroups?
5) Is $(\mathbb{Z}, +)$ a cyclic group? If yes, find all generators.
6) Find order of each element of group $(G, \cdot)$, where $G = \{1, -1, i, -i\}$.
7) Define simple group.
8) State second isomorphism theorem.
9) Define coset in a group.
10) Express the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 5 & 6 & 1 & 4 \end{pmatrix}$ as a product of disjoint cycles.
11) Is the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$ commutative?
12) What is signature of the permutation?

Q. 4 Attempt any FOUR **(32)**

1) Prove that $(\mathbb{Z}_5^*, \cdot)$ is an abelian group.
2) Prove that every subgroup of cyclic group is cyclic.
3) State and prove Lagrange's theorem for a finite group.
4) State and prove Fermat's theorem.
5) Prove that a homomorphism f is one-one iff $Kerf = \{e\}$?
6) State and prove First isomorphism theorem.
7) Define cycle, Even and odd permutations and transpositions.
8) State and prove Cayley's theorem.

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