[150] SEAT No.________

SARDAR PATEL UNIVERSITY(B.Sc. Sem.6 Examination)
MATHEMATICS - US06CMTH22 – Ring Theory
5th April 2022, Tuesday

No of printed pages :02

Time: 03:00 TO 05:00 p.m. Maximum Marks: 70

Note: Figures to the right indicates the full marks.

Q.1 Answer the following by selecting the correct choice from the given options. [10]

1. ________ is a ring with no unit element.
   (a) Z (b) $Z-\{0\}$ (c) $Z-\{0,1\}$ (d) $2Z$
2. In a Boolean ring, $a+a=$ ________
   (a) 0 (b) a^2 (c) a (d) 1
3. ______ is not an integral domain.
   (a) Z_4 (b) Z_5 (c) Q (d) C
4. Every element of Equivalence class can be expressed as _______, $a,b\in R, b\neq 0$
   (a) $a\sim b$ (b) ab^{-1} (c) $\overline{(a,b)}$ (d) $\widetilde{(a,b)}$
5. ________ is a smallest field containing R.
   (a) Z (b) Q (c) F (d) C
6. A field of Ch.0 contains a __________ field.
   (a) integer (b) rational (c) special (d) prime
7. In $R=Z+iZ$, $(2,-1+5i)$=________.
   (a) $1+i$ (b) $1-i$ (c) 1 (d) i
8. $1+i$ is _____ in $Z+iZ$
   (a) unit (b) regular element (c) irreducible (d) reducible
9. If p is prime, and $n>1$, $\sqrt[n]{p}$ is_______.
   (a) irrational (b) prime (c) rational (d) composite
10. If $f(x)=3x^3-3x^2+9x+6\in Z[x]$ then $C(f)=$_______.
    (a) 0 (b) 1 (c) 2 (d) 3

Q.2 Answer the following.(True/False) [08]

1. The ring of real quaternions is commutative.
2. In a field every non-zero element is regular.
3. Every prime ideal is maximal.
4. If I and J are ideals in R then $I\cup J$ is an ideal in R.
5. In a ring every irreducible element is prime.
6. Any associate of an irreducible element is also irreducible.
7. $R[x]$ is a field.
8. $Z[x]$ is a unique factorisation domain.

P.T.O.

Q.3 Answer ANY TEN of the following. [20]

1 Prove or disprove that Z_6 is an integral domain.
2 Give an example of a Boolean ring. Justify your example.
3 Define: embedding
4 If R is a finite commutative ring then prove that every prime ideal of R is a maximal ideal.
5 Prove that every field is a simple ring.
6 Define: Left Ideal
7 If $R = \{a + b\sqrt{-5} \,/ a, b \in Z\}$ and $a = 1 + 2\sqrt{-5}$ and $b = 3$ then find (a, b).
8 Show that the gcd of two elements if it exists is unique up to units.
9 Define: Euclidean domain
10 If F is a field then prove that $F[x]$ is a principal ideal domain.
11 Find a root of $f(x) = x^2 - 3x + 3 - i$ in $R = Z + iZ$
12 Define: Multiple root in polynomial ring

Q-4 Answer ANY FOUR of the following. (32)

1 Let R be the set of all subsets of X. Define $+$ and $\cdot$ on R by $A + B = (A - B) \cup (B - A)$ and $A \cdot B = A \cap B$. Then show that R is a ring. Also show that R is a commutative ring with unit element.
2 Let R be a ring. Then prove that
(i) $a \cdot 0 = 0 = 0 \cdot a, \forall a \in R$
(ii) $a(-b) = (-a)b = -(ab), \ \forall a, b \in R$
(iii) $(-a)(-b) = ab, \ \forall a, b \in R$
3 State and prove First Isomorphism theorem.
4 Prove that P is a prime ideal of Z iff either $P = \{0\}$ or $P = pZ$ for some prime p.
5 Show that the ring of Gaussian integers is an Euclidean domain.
6 Let R be an Euclidean domain. Then prove that any $a \in R$ which is not a unit can be expressed as a product of irreducible elements.
7 If R is a Euclidean domain or a principal ideal domain, then prove that R is a unique factorisation domain.
8 Show that S_α is a ring homomorphism.