

[150] SEAT No. _____



SARDAR PATEL UNIVERSITY (B.Sc. Sem.6 Examination)

MATHEMATICS - US06CMTH22 - Ring Theory

5th April 2022, Tuesday

Time: 03:00 TO 05:00 p.m.

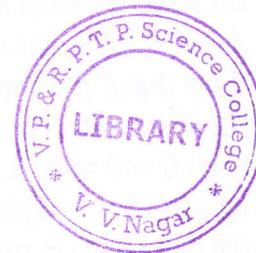
Maximum Marks: 70

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Note: Figures to the right indicates the full marks.

Q.1 Answer the following by selecting the correct choice from the given options. [10]

- 1 _____ is a ring with no unit element.
(a) Z (b) $Z - \{0\}$ (c) $Z - \{0,1\}$ (d) $2Z$
- 2 In a Boolean ring, $a + a =$ _____
(a) 0 (b) a^2 (c) a (d) 1
- 3 _____ is not an integral domain.
(a) Z_4 (b) Z_5 (c) Q (d) C
- 4 Every element of Equivalence class can be expressed as _____, $a, b \in R, b \neq 0$
(a) $a \sim b$ (b) ab^{-1} (c) (a, b) (d) $(\overline{a}, \overline{b})$
- 5 _____ is a smallest field containing R .
(a) Z (b) Q (c) F (d) C
- 6 A field of Ch.0 contains a _____ field.
(a) integer (b) rational (c) special (d) prime
- 7 In $R = Z + iZ$, $(2, -1 + 5i) =$ _____.
(a) $1 + i$ (b) $1 - i$ (c) 1 (d) i
- 8 $1 + i$ is _____ in $Z + iZ$
(a) unit (b) regular element (c) irreducible (d) reducible
- 9 If p is prime, and $n > 1$, $\sqrt[n]{p}$ is _____.
(a) irrational (b) prime (c) rational (d) composite
- 10 If $f(x) = 3x^3 - 3x^2 + 9x + 6 \in Z[x]$ then $C(f) =$ _____.
(a) 0 (b) 1 (c) 2 (d) 3



Q.2 Answer the following. (True/False) [08]

- 1 The ring of real quaternions is commutative.
- 2 In a field every non-zero element is regular.
- 3 Every prime ideal is maximal.
- 4 If I and J are ideals in R then $I \cup J$ is an ideal in R .
- 5 In a ring every irreducible element is prime.
- 6 Any associate of an irreducible element is also irreducible.
- 7 $R[x]$ is a field.
- 8 $Z[x]$ is a unique factorisation domain.

P.T.O.



Q.3 Answer ANY TEN of the following.

[20]

- 1 Prove or disprove that Z_6 is an integral domain.
- 2 Give an example of a Boolean ring. Justify your example.
- 3 Define: embedding
- 4 If R is a finite commutative ring then prove that every prime ideal of R is a maximal ideal.
- 5 Prove that every field is a simple ring.
- 6 Define: Left Ideal
- 7 If $R = \{a + b\sqrt{-5} / a, b \in Z\}$ and $a = 1 + 2\sqrt{-5}$ and $b = 3$ then find (a, b) .
- 8 Show that the gcd of two elements if it exists is unique up to units.
- 9 Define: Euclidean domain
- 10 If F is a field then prove that $F[x]$ is a principal ideal domain.
- 11 Find a root of $f(x) = x^2 - 3x + 3 - i$ in $R = Z + iZ$
- 12 Define: Multiple root in polynomial ring

Q-4 Answer ANY FOUR of the following.

[32]

- 1 Let R be the set of all subsets of X . Define $+$ and \cdot on R by $A + B = (A - B) \cup (B - A)$ and $A \cdot B = A \cap B$. Then show that R is a ring. Also show that R is a commutative ring with unit element.
- 2 Let R be a ring. Then prove that
 - (i) $a \cdot 0 = 0 = 0 \cdot a, \forall a \in R$
 - (ii) $a(-b) = (-a)b = -(ab), \forall a, b \in R$
 - (iii) $(-a)(-b) = ab, \forall a, b \in R$
- 3 State and prove First Isomorphism theorem.
- 4 Prove that P is a prime ideal of Z iff either $P = \{0\}$ or $P = pZ$ for some prime p .
- 5 Show that the ring of Gaussian integers is an Euclidean domain.
- 6 Let R be an Euclidean domain. Then prove that any $a \in R$ which is not a unit can be expressed as a product of irreducible elements.
- 7 If R is a Euclidean domain or a principal ideal domain, then prove that R is a unique factorisation domain.
- 8 Show that S_α is a ring homomorphism.

