[167] SEAT No.__________ No. of Printed Pages : 02

**SARDAR PATEL UNIVERSITY**

**B.Sc. SEM :6, APRIL : 2022**

**MATHEMATICS, US06CMTH24**

**(Riemann Integration and Series of Functions)** **Max. Marks : 70**

**Date :07/04/2022**, Thursday **Time: 3.00 to 5.00** p.m.

**Q.1 Choose the correct option for each of the following.** **[10]**



(1) If P is a partition of [a,b] then

(a) a ∈ P, but b ∉ P (b) a ∉ P, but b ∉ P (c) a ∉ P, but b ∈ P (d) a ∈ P, but b ∈ P

(2) $\int_a^{-b} f(x)dx =$ ........

(a) $Sup(U(P,f))$ (b) $Sup(L(P,f))$ (c) $Inf(U(P,f))$ (d) $Inf(L(P,f))$

(3) If $\mu$ is a mesh of the partition $P = \{x_0, x_1, \dots, x_n\}$ for [a,b] then..... for every i= 1,2,...,n

(a) $\Delta xi = \mu$ (b) $\Delta xi < \mu$ (c) $\Delta xi > \mu$ (d) $\Delta xi \le \mu$

(4) If $P, P^*$ are any two partitions of [a,b] then $|S(P,f) - S(P^*,f)|$ ....... $\varepsilon$

(a) $=$ (b) $<$ (c) $>$ (d) $\geq$

(5) A function $f$ cannot be integrable over $[a,b]$, if it is.....

(a) Increasing over $[a,b]$ (b) Decreasing over $[a,b]$ (c) Continuous over $[a,b]$ (d) none of these

(6) The Reimann Sum of a bounded $f$ on $[a,b]$ w.r.to a Partiton $P$ is denoted by .........

(a) $U(P,f)$ (b) $L(P,f)$ (c) $S(P,f)$ (d) none of these

(7) $\int_0^1 \frac{sinx}{x} dx$ is ........ integral.

(a) finite (b) infinite (c) proper (d) improper

(8) $\int_0^1 \frac{logx}{\sqrt{x}} dx$ is .......

(a) convergent (b) divergent (c) infinite (d) none of these

(9) The values of series $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} +$......... = .......

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{-3}{2}$ (d) none of these

(10) The sequence $\{\frac{nx}{1+n^3x^2}\}$ converges uniformly to ....... , for $0 \le x \le 1$.

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) none of these

**Q.2 Do as directed.** **[8]**

(1) True or False : $\int_{-a}^{b} f(x)\,dx \le \int_a^{-b} f(x)\,dx$.

(2) If $P = \left\{-2, -1, \frac{-1}{2}, 0, \frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2}, 2\right\}$ is a partition of [-2,2] then $\mu(P) =$ ...4...

(3) $\int_0^3 [x]dx =$ 4........

(4) True or False : A bounded function $f$ having a finite number of points of discontinuity on [a,b] is always integrable on [a,b].



(P.T.O.)

(5) True or False : $\int_0^2 \frac{1}{2x-x^2} dx$ integral diverges.

(6) $\int_0^{\frac{\pi}{2}} logsinx\ dx$ = .........

(7) True or False : The series $\sum_1^{\infty} \frac{x}{n(1+nx^2)}$ converges uniformly for all real $x$.

(8) True or False : The sequence $\{f_n\}$, where $f_n(x) = nxe^{-nx^2}, n = 1,2,3$ .... Converges pointwise to zero on [0,1].

**Q.3 Attempt any TEN.** **[20]**

(1) For a bounded function $f(x) = x^2 , x \in [-1,2]$ and a partition $P = \{-1, -\frac{1}{2}, 0,1,2\}$ of [-1,2] then find $L(P, f)$.

(2) Define : The Upper Riemann sum of a bounded function.

(3) In usual notation , prove that $m(b-a) \leq \int_a^b f(x)dx \leq M(b-a), a \leq b$.

(4) Define : Riemann Integral (Second form).

(5) State First mean value theorem of differential Calculus.

(6) State the second Fundamental theorem of integral calculus.

(7) Define : Improper integral.

(8) Examine the convergence of $\int_0^1 \frac{dx}{x^2}$.

(9) Examine the convergence of $\int_0^1 \frac{dx}{\sqrt{1-x^3}}$.

(10) Define : Uniform convergence on an interval.

(11) State Able's test.

(12) Prove that the sequence $\{f_n\}$, where $f_n(x) = \frac{1}{x+n}$ is uniformly convergent in [0,b], b> 0.

**Q.4 Attempt any FOUR.** **[32]**

(1) State and Prove Darboux's theorem.

(2) If $f$ is bounded and integrable function on [a,b] and $c$ is any constant then prove that $cf$ Is also integrable on [a,b] and $\int_a^b cf\ dx = c \int_a^b f\ dx$.

(3) If a function $f$ is continuous on [a,b] then prove that $f$ is integrable on [a,b].

(4) State and Prove the First Fundamental theorem of integral calculus.

(5) State and Prove the comparison test-II for convergence of an improper integral.

(6) State and Prove that Cauchy's Test for convergence of improper integral.

(7) State and Prove Weistrass's M – Test.

(8) Let $\{f_n\}$ be a sequence of functions such that $\lim_{n\to\infty} f_n(x) = f(x),\ x \in [a,b]$ and

$M_n = \sup_{x\in[a,b]} |f_n(x) - f(x)|$ then prove that $f_n \to f$ uniformly on [a,b] if and only if

$M_n \to 0$ as $n \to \infty$.

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