V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2013-14
F.Y.B.Sc. : Semester - I (CBCS)

Subject : Mathematics   US01CMTH02   Max. Marks : 30
Calculus and Differential Equations
Date: 08/10/2013   Timing: 11.00 am - 12.00pm

Instructions : (1) This question paper contains FIVE QUESTIONS
(2) The figures to the right side indicate full marks of the corresponding question/s
(3) The symbols used in the paper have their usual meaning, unless specified

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Q: 1. Answer the following by choosing correct answers from given choices. 6

[1] If $y = e^{4x} + e^{2x}$ then $y_n =$

[A] $e^{2x}(2^n e^{2x} + 1)$ [B] $2^n e^{2x}(2^n e^{2x} + 1)$ [C] $e^{2x}(2^n e^{2x} - 1)$ [D] none

[2] For $y = a^{mx}$, $y_n =$

[A] $n^m (\log a)^n a^{mx}$ [B] $m^n (\log a)^n a^{mx}$ [C] $m^n a^{mx}$ [D] $n^m a^{mx}$

[3] At a point on a curve, with non zero curvature, the radius of curvature and the curvature are

[A] Additive inverses of each other
[B] Multiplicative inverses of each other
[C] equal
[D] none

[4] For $r = f(\theta)$ which of the following can be used to measure radius of curvature

[A] $\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$ [B] $\frac{r}{r_1}$ [C] $\frac{(r^2 + r_1^2)^{\frac{3}{2}}}{r^2 + 2r_1^2 - rr_2}$ [D] $\sqrt{1 + \left(\frac{dr}{d\theta}\right)^2}$

[5] The degree of the homogeneous function $f(x, y) = \frac{x^2 + y^2}{x - y}$ is

[A] 0 [B] 1 [C] 2 [D] 3

[6] Euler's theorem requires a homogeneous function

[A] to be continuous on its domain
[B] to possess second order partial derivatives
[C] to possess first order partial derivatives
[D] none

Q: 2. Answer any THREE of the following. 6

[1] If $y = \cos 3x$ then find y_4

[2] If $y = e^{mx}$, then show that $y_n = m^n e^{mx}$

[3] Let $y = f(x)$ be a cartesian representation of a curve C. Then prove that the length of arc of C between two points A and B corresponding to the x-coordinates a and b respectively, is given by

$$arc\widehat{AB} = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx.$$

[4] Define the terms : (i) Radius of Curvature (ii) Intrinsic Equation

[5] Verify Euler's theorem for the function $z = x^2y - xy^2$

[6] Find $\frac{dy}{dx}$ when $x^y = y^x$

Q: 3. If the angle between radius vector and tangent at a point on a polar curve is ϕ then prove that $\tan\phi = \frac{r}{\left(\frac{dr}{d\theta}\right)}$ 6

OR

Q: 3. If $y = (x - \sqrt{4 + x^2})^m$,then find $y_n(0)$ 6

Q: 4. Let $r = f(\theta)$ be a polar form of a curve with a point P on it. Then prove that the radius of curvature at P is given by

$$\rho = \frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - rr_2}.$$

where $r_1 = f'(\theta)$ and $r_2 = f''(\theta)$ 6

OR

Q: 4 [A] Find the entire length of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ 3

[B] Find the length of arc of the parabola $y^2 = 4ax$, $(a > 0)$, measured from the vertex to one extremity of its latus rectum 3

Q: 5. Let $z = f(x, y)$ be a real valued function defined on $E \subset R^2$. Suppose that f is a homogeneous function of degree n and that all the second order partial derivatives of f exist and are continuous. Then prove that

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z.$$

6

OR

Q: 5 [A] If $u = \sin^{-1}(\frac{x^2y^2}{x+y})$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\tan u$ 3

[B] If $H = f(2x - 3y; 3y - 4z, 4z - 2x)$, then prove that

$$\frac{1}{2}\frac{\partial H}{\partial x} + \frac{1}{3}\frac{\partial H}{\partial y} + \frac{1}{4}\frac{\partial H}{\partial z} = 0.$$

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