V.P.& R.P.T.P.Science College.Vallabh Vidyanagar.
Internal Test
B.Sc. Semester II
US02CMTH01 (Analytic Solid Geometry)
Saturday , 15^{th} March 2014
11.00 a.m. to 12.00 p.m.

Maximum Marks: 30

Que.1 Answer the following (Any three) 6

(1) Prove that a sphere with centre (α, β, γ) and radius a is given by $(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2=a^2$.

(2) Find the equations of the tangent plane to the sphere $x^2+y^2+z^2+2x+4y+6z-24=0$ at (1,1,2).

(3) Show that $Ax^2+By^2+Cz^2=D$ represents an elliptic hyperboloid of one sheet if one coefficient is negative and $D>0$.

(4) Plot the spherical point $(2, 7\pi/4, \pi/6)$.

(5) Find the points of intersection of the line $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$ and the cone $f(x,y,z) \equiv ax^2+by^2+cz^2+2fyz+2gzx+2hxy+2ux+2vy+2wz=0$.

(6) Find the equation of cone with vertex at the origin and which passes through the curve $ax^2+by^2=2z$; $lx+my+nz=p$.

Que.2 (a) Find the equation of the spheres which pass through the given circle $x^2+y^2+z^2-4x-y+3z+12=0$; $2x+3y-7z=10$ and touch the plane $x-2y+2z=1$. 6

(b) Find equation of sphere with centre at (2,-1,0) and passing through (1,-1,2). 2

OR

Que.2 (a) Let two spheres be given by 5
$S_1 \equiv x^2+y^2+z^2+2u_1x+2v_1y+2w_1z+d_1=0$;
$S_2 \equiv x^2+y^2+z^2+2u_2x+2v_2y+2w_2z+d_2=0$.

Then prove that $S_1+\lambda S_2=0$,where $\lambda \in \mathbb{R}$, $\lambda \neq -1$, represents a family of spheres passing through the intersection of the spheres $S_1=0$ and $S_2=0$.

(b) Show that the following pair of spheres touch each other
$x^2+y^2+z^2=64$; $x^2+y^2+z^2-12x+4y-6z+48=0$. 3

Que.3 (a) Identify , describe and sketch the surface $\frac{x^2}{a^2}+\frac{y^2}{b^2}=cz$; $(c>0)$. 5

(b) Find Jacobian of Cartesian co-ordinates with respect to Cylindrical co-ordinates. 3

OR

Que.3 (a) By a proper choice of axes , prove that the Cartesian coordinates (x , y , z) of a point can be expressed in terms of spherical polar coordinates (ρ, θ, ϕ) as $x = \rho \sin\phi \cos\theta$, $y = \rho \sin\phi \sin\theta$, $z = \rho \cos\phi$. 4

(b) Identify and describe the surface $\frac{x^2}{9} - \frac{y^2}{16} - \frac{z^2}{9} = 1$. 4

Que.4 (a) Prove that the cone $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ admits of sets of three mutually perpendicular generators iff $a + b + c = 0$. 5

(b) Find the equation of cone whose vertex is (α, β, γ) and base is $ax^2 + by^2 = 1$; $z = 0$. 3

OR

Que.4 (a) If the section of a cone whose vertex is P and guiding curve is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; $z = 0$ by the plane x=0 is a rectangular hyperbola .Show that the locus of P is $\frac{x^2}{a^2} + \frac{y^2 + z^2}{b^2} = 1$. 5

(b) If $F(x, y, z) \equiv ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2ux + 2vy + 2wz + d = 0$ represents a cone , then the co-ordinates of its vertex satisfy the equation $F_x = F_y = F_z = F_t = 0$ where t is used to make F(x,y,z) homogeneous and put t equal to unity after differentiation. 3

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