

V.P.& R.P.T.P.Science College,Vallabh Vidyanagar.

Internal Test

B.Sc. Semester II

US02CMTH01 (Analytic Solid Geometry)

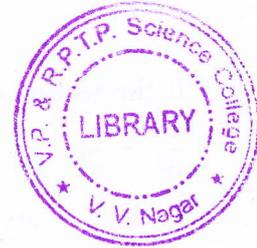
Saturday , 15th March 2014

11.00 a.m. to 12.00 p.m.

Maximum Marks: 30

Que.1 Answer the following (Any three)

- (1) Prove that a sphere with centre (α, β, γ) and radius a is given by $(x - \alpha)^2 + (y - \beta)^2 + (z - \gamma)^2 = a^2$.
- (2) Find the equations of the tangent plane to the sphere $x^2 + y^2 + z^2 + 2x + 4y + 6z - 24 = 0$ at $(1,1,2)$.
- (3) Show that $Ax^2 + By^2 + Cz^2 = D$ represents an elliptic hyperboloid of one sheet if one coefficient is negative and $D > 0$.
- (4) Plot the spherical point $(2, 7\pi/4, \pi/6)$.
- (5) Find the points of intersection of the line $\frac{x - \alpha}{l} = \frac{y - \beta}{m} = \frac{z - \gamma}{n}$ and the cone $f(x, y, z) \equiv ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2ux + 2vy + 2wz = 0$.
- (6) Find the equation of cone with vertex at the origin and which passes through the curve $ax^2 + by^2 = 2z$; $lx + my + nz = p$.



- Que.2 (a) Find the equation of the spheres which pass through the given circle $x^2 + y^2 + z^2 - 4x - y + 3z + 12 = 0$; $2x + 3y - 7z = 10$ and touch the plane $x - 2y + 2z = 1$. 6
- (b) Find equation of sphere with centre at $(2,-1,0)$ and passing through $(1,-1,2)$. 2

OR

- Que.2 (a) Let two spheres be given by 5
 $S_1 \equiv x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$;
 $S_2 \equiv x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0$.

Then prove that $S_1 + \lambda S_2 = 0$, where $\lambda \in \mathbb{R}$, $\lambda \neq -1$, represents a family of spheres passing through the intersection of the spheres $S_1 = 0$ and $S_2 = 0$.

- (b) Show that the following pair of spheres touch each other 3
 $x^2 + y^2 + z^2 = 64$; $x^2 + y^2 + z^2 - 12x + 4y - 6z + 48 = 0$.

- Que.3 (a) Identify , describe and sketch the surface $\frac{x^2}{a^2} + \frac{y^2}{b^2} = cz$; $(c > 0)$. 5
- (b) Find Jacobian of Cartesian co-ordinates with respect to Cylindrical co-ordinates. 3

OR

Que.3 (a) By a proper choice of axes , prove that the Cartesian coordinates (x , y , z) of a point can be expressed in terms of spherical polar coordinates (ρ, θ, ϕ) as
 $x = \rho \sin \phi \cos \theta$, $y = \rho \sin \phi \sin \theta$, $z = \rho \cos \phi$. 4

(b) Identify and describe the surface $\frac{x^2}{9} - \frac{y^2}{16} - \frac{z^2}{9} = 1$. 4

Que.4 (a) Prove that the cone $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ admits of sets of three mutually perpendicular generators iff $a + b + c = 0$. 5

(b) Find the equation of cone whose vertex is (α, β, γ) and base is $ax^2 + by^2 = 1$; $z = 0$. 3

OR

Que.4 (a) If the section of a cone whose vertex is P and guiding curve is the ellipse 5

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; $z = 0$ by the plane $x=0$ is a rectangular hyperbola .Show that the

locus of P is $\frac{x^2}{a^2} + \frac{y^2 + z^2}{b^2} = 1$.

(b) If $F(x, y, z) \equiv ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2ux + 2vy + 2wz + d = 0$ represents a cone , then the co-ordinates of its vertex satisfy the equation $F_x = F_y = F_z = F_t = 0$ where t is used to make F(x,y,z) homogeneous and put t equal to unity after differentiation. 3

