V.P. & R.P.T.P. Science College,V.V.Nagar

Internal Test: 2013-14

F.Y.B.Sc. : Semester - II (CBCS)

Subject : Mathematics US02CMTH02 Max. Marks : 30

Matrix Algebra and Differential Equations

Date: 18/03/2014 Timing: 11.00 am - 12.00pm

Q: 1. Answer any THREE of the following. **6**

[1] Define : (i) Column Matrix (ii) Unit Matrix

[2] If A and B both are symmetric, then prove that AB is symmetric *iff A and B commute.*

[3] Find the characteristic equation of $\begin{bmatrix} 2 & -7 & 1 \\ -1 & 2 & 1 \\ 2 & 2 & 1 \end{bmatrix}$

[4] Find the characteristic roots of $\begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}$

[5] Find the complementary function of $(D^2 - 8D + 16)y = e^{2x}$

[6] Find $\dfrac{1}{(D^6 + D^2 + 1)} \sin 2x$

Q: 2 [A] Prove that every square matrix can be expressed in one and only one way as a sum of a symmetric and a skew-symmetric matrix. **4**

[B] For $A = \begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix}$, where $l = \dfrac{1}{\sqrt{2}}$, $m = \dfrac{1}{\sqrt{6}}$ and $n = \dfrac{1}{\sqrt{3}}$ show that $AA' = I$ **4**

OR

Q: 2 [A] Prove that every Hermitian matrix over $\mathbb{C}$ can be uniquely expressed as $P + iQ$, where P and Q are real symmetric and skew-symmetric matrices respectively **4**

[B] If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then show that $A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$ where k is any positive integer **4**

Q: 3 [A] If S is a real skew-symmetric matrix then prove that $I - S$ is non-singular and the matrix $A = (I + S)(I - S)^{-1}$ is orthogonal **4**

[B] Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and verify that it is satisfied by A and hence obtain A^{-1} 4

OR

Q: 3 [A] State and prove *Cayley-Hamilton theorem* 4

[B] Find characteristic roots and any one of the characteristic vectors of : $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ 4

Q: 4 [A] Prove that the differential equation

$$\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} + \cdots + a_n y = 0$$

with constant coefficients always admits the general solution, when the roots of auxiliary equation are all real. 8

OR

Q: 4 [A] Solve : $(D^3 - 1)y = (e^x - 1)^2$ 4

[B] Solve : $(D^3 - 4D^2 + 5D - 2)y = 0$ 4