V.P. & R.P.T.P. Science College,V.V.Nagar

Internal Test: 2014-15

Subject : Mathematics US01CMTH02 Max. Marks : 25

Calculus and Differential Equations

Date: 03/12/2014 Timing: 11:00 am - 12:00 pm

Instructions : (1) This question paper contains FIVE QUESTIONS

(2) The figures to the right side indicate full marks of the corresponding question/s

(3) The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices. 3

[1] Leibniz's theorem can be applied to a product of two functions which are sufficiently many times

[A] differentiable [B] continuous [C] integrable [D] none of these

[2] At a point on a curve, with non zero curvature, the radius of curvature and the curvature are

[A] Additive inverses of each other
[B] Multiplicative inverses of each other
[C] equal
[D] none

[3] The degree of the homogeneous function $f(x, y) = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{x}{y}\right)$ is

[A] 0 [B] 1 [C] -1 [D] undefined

Q: 2. Answer any TWO of the following. 4

[1] If $y = \sin 4x$ then find y_4

[2] Prove that if ρ is the radius of curvature at any point P of the parabola $y^2 = 4ax$ and S is its focus then prove that $\rho^2 \propto SP^3$

[3] Verify Euler's theorem for the function $z = \sin^{-1}\frac{x}{y}$

Q: 3. State and prove Leibniz's theorem 6

OR

Q: 3 [A] Find y_n for $y = e^{2x}\cos x \sin^2 2x$ 3

[B] Find the angle between radius vector and tangent at a point on the curve : $r^m = a^m(\cos m\theta + \sin m\theta)$ 3

Q: 4. For a polar equation $r = f(\theta)$ of a curve,prove that

$$\frac{ds}{d\theta} = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}.$$

6

OR

Q: 4. For the curve $r = a(1 - \cos\theta)$, prove that $\rho^2 \propto r$. Also prove that if ρ_1 and ρ_2 are radii of the curvature at the ends of a chord through the pole, $\rho_1^2 + \rho_2^2 = \frac{16a^2}{9}$ 6

Q: 5. Define a homogeneous function and state and prove Euler's theorem for function of three variable 6

OR

Q: 5 [A] If $H = f(2x - 3y, 3y - 4z, 4z - 2x)$, then prove that

$$\frac{1}{2}\frac{\partial H}{\partial x} + \frac{1}{3}\frac{\partial H}{\partial y} + \frac{1}{4}\frac{\partial H}{\partial z} = 0.$$

3

[B] If $u = \sin^{-1}(\frac{x^2y^2}{x+y})$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\tan u$ 3