# V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2015-16

Subject : Mathematics US02CMTH02 Max. Marks : 25

Matrix Algebra and Differential Equations

Date: 19/03/2016 Timing: 01.30 pm - 2.30 pm

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Instructions : (1) This question paper contains FOUR questions.
(2) The figures to the right side indicate full marks of the corresponding question/s
(3) The symbols used in the paper have their usual meaning, unless specified.

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**Q: 1.** Answer the following by choosing correct answers from given choices. **3**

[ 1] For a square matrix $A$ over R the matrix $A - A'$ is
[A] symmetric [B] skew symmetric [C] hermitian [D] skew hermitian

[ 2] If 3 is a characterstic root of $A$ then
[A] $|I + 3A|=0$ [B] $|I - 3A|=0$ [C] $|A + 3I|=0$ [D] $|A - 3I|=0$



[ 3] For a square matrix $A$ if $AX = 2X$, $X \neq O$ then
[A] X is characteristic root of A corresponding to 2
[B] A is characteristic root of X corresponding to 2
[C] A is characteristic vector of X corresponding to 2
[D] X is a characteristic vector of A corresponding to 2

**Q: 2.** Answer any TWO of the following. **4**

[ 1] Define : (i) Skew-Hermitian Matrix (ii) Scalar Matrix

[ 2] Determine whether the matrix $\begin{bmatrix} 7-4i & 5-i & 1 \\ 4i-1 & 6+i & 2-i \\ 3 & i-4 & 9+4i \end{bmatrix}$ is Skew-Hermitian or not.

[ 3] Find the characteristic equation of $\begin{bmatrix} 4 & 1 & -2 \\ 1 & 0 & 2 \\ 0 & -1 & 5 \end{bmatrix}$

[ 4] Find the transpose of $D = \begin{bmatrix} 7 & 3 \\ 1 & 2 \end{bmatrix}$ and determine whether the transpose is an orthogonal matrix or not.

**Q: 3** [A] Prove that every square matrix can be expressed in one and only one way as a sum of a Hermitian and a skew-Hermitian matrix. **5**

[ B] For $A = \begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix}$, where $l = \frac{1}{\sqrt{2}}$, $m = \frac{1}{\sqrt{6}}$ and $n = \frac{1}{\sqrt{3}}$ show that $AA' = I$ **4**

OR

**Q: 3 [A]** State and prove the *reversal law* for the transpose of product of matrices and deduce the reversal law for conjugate transpose of product of matrices. **5**

**[ B]** If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, then find the values of $\alpha$ and $\beta$ such that $(\alpha I + \beta A)^2 = A$ **4**

**Q: 4 [A]** If $S$ is a real skew-symmetric matrix then prove that $I - S$ is non-singular and the matrix $A = (I + S)(I - S)^{-1}$ is orthogonal **5**

**[ B]** Verify *Cayley-Hamilton* theorem for the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 3 & -3 & 2 \\ 1 & 1 & -1 \end{bmatrix}$. Hence find its inverse if possible **4**

OR

**Q: 4 [A]** State and prove *Cayley-Hamilton theorem* **5**

**[ B]** Find eigen values and any one of the eigen vectors of $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ **4**