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Internal Test: 2017-18
Subject: Mathematics US01CMTH02 Max. Marks: 25

Calculus and Differential Equation

Date: 11/10/2017 Timing: 01:30 pm - 02:30 pm

3 Q: 1. Answer the following by choosing correct answers from given choices. [1] If $y = e^{3x} \cos 2x$ then $y_n =$ [A] $13^{\frac{n}{2}}e^{3x} \cos(2x + n \tan^{-1}\frac{1}{3})$ [B] $13^{\frac{n}{2}}e^{3x} \cos(2x + n \tan^{-1}\frac{3}{2})$ [C] $13^{\frac{n}{2}}e^{3x}\cos(2x + n \tan^{-1}3)$ [D] $13^{\frac{n}{2}}e^{3x}\cos(2x + n \tan^{-1}\frac{2}{3})$ [2] Intrinsic equation of a curve involves [A] cartesian coordinates only [B] polar coordinates only [C] parameteric coordinates only [D] none of these [3] If a function y of x be implicitly described by f(x,y) = c, where c is a constant then [A] $\frac{dy}{dx} = -\frac{f_y}{f_x}$ [B] $\frac{dy}{dx} = \frac{f_y}{f_x}$ [C] $\frac{dy}{dx} = \frac{f_x}{f_y}$ [D] $\frac{dy}{dx} = -\frac{f_x}{f_y}$ 4 Q: 2. Answer ANY TWO of the following. [1] If $y = \log(2x - 1)$ then find y_4 [2] Define: (i) Intrinsic Equation (ii) Rectification [3] Verify Euler's theorem for the function $z = \sin^{-1} \frac{x^2}{x^2}$ 3 Q: 3 [A] State and prove Leibniz's theorem [B] If $y = e^{ax} \cos(bx + c)$, then prove that $y_n = r^n e^{ax} \cos(bx + c + n\varphi)$, where $r = \sqrt{a^2 + b^2}$, $\varphi = \tan^{-1}\left(\frac{b}{a}\right)$ 3

Q: 3 [A] Let
$$y = (x^2 - 2)^m$$
. Find the value of m such that $(x^2 - 2)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$.

[B] Find the angle between radius vector and tangent at a point on the curve
$$r^m = a^m(\cos m\theta + \sin m\theta)$$

Q: 4 [A] Prove that if
$$\rho$$
 is the radius of curvature at any point P of the parabola $y^2 = 4ax$ and S is its focus then prove that $\rho^2 \propto SP^3$

[B] For the curve
$$r = a(1 - \cos \theta)$$
, prove that $\rho^2 \propto r$. Also prove that if ρ_1 and ρ_2 are radii of the curvature at the ends of a chord through the pole, $\rho_1^2 + \rho_2^2 = \frac{16a^2}{9}$

Define Radius of curvature and let $r = f(\theta)$ be a polar form of a curve with a point P on it. Then prove that the radius of curvature at P is given by

$$\rho = \frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - rr_2},$$

where $r_1 = f'(\theta)$ and $r_2 = f''(\theta)$

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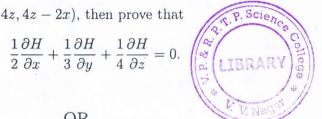
Q: 5 [A] Let z = f(x,y) be a real valued function defined on $E \subset \mathbb{R}^2$. Suppose that f is a homogeneous function of degree n and that all the second order partial derivatives of f exist and are continuous. Then prove that

$$x^{2} \frac{\partial^{2} z}{\partial x^{2}} + 2xy \frac{\partial^{2} z}{\partial x \partial y} + y^{2} \frac{\partial^{2} z}{\partial y^{2}} = n(n-1)z.$$

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[B] If H = f(2x - 3y, 3y - 4z, 4z - 2x), then prove that

$$\frac{1}{2}\frac{\partial H}{\partial x} + \frac{1}{3}\frac{\partial H}{\partial y} + \frac{1}{4}\frac{\partial H}{\partial z} = 0$$



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OR

Q: 5 [A] Let a function y of x be implicitly described by f(x,y) = c. Then prove that

$$(1) \frac{dy}{dx} = -\frac{f_x}{f_y}$$

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(2) $\frac{d^2y}{dx^2} = -\frac{f_{xx}(f_y)^2 - 2f_{xy}f_xf_y + f_{yy}(f_x)^2}{(f_y)^3}$

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B If A, B and C are angles of a $\triangle ABC$ such that $\sin^2 A + \sin^2 B + \sin^2 C = K$, a constant, then prove that

$$\frac{dB}{dC} = \frac{\tan C - \tan A}{\tan A - \tan B}$$

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