V.P.& R.P.T.P.Science College , Vallabh Vidyanagar.
B.Sc.(Semester - II) Internal Test
US02CMTH21 (Algebra)

Date. 9/3/2019 : Saturday 12.30 p.m. to 2.30 p.m. Maximum Marks: 50

Que.1 Fill in the blanks. 8

(1) The modulus of $\dfrac{(3-\sqrt{2}\,i)^2}{1+2i}$ is ___________

(a) $\dfrac{11}{\sqrt{5}}$ (b) $\dfrac{11}{5}$ (c) $\dfrac{7}{\sqrt{5}}$ (d) $\dfrac{13}{\sqrt{5}}$

(2) General value of $\log(-i) =$___________.

(a) $i\left(n-\dfrac{1}{2}\right)\pi$ (b) $i\left(2n-\dfrac{1}{2}\right)$ (c) $\left(2n-\dfrac{1}{2}\right)\pi$ (d) $i\left(2n-\dfrac{1}{2}\right)\pi$

(3) If any set $X \sim N$ then X is said to be ___________ set.
(a) finite (b) denumerable (c) empty (d) oneone

(4) Non zero matrices A and B are called divisor of zero if ___________.
(a) $A = O, B \neq O$ (b) $AB \neq O$ (c) $AB = O$ (d) None

(5) If a matrix A has non-zero minor of order r then ___________.
(a) $\rho(A) = r$ (b) $\rho(A) \leq r$ (c) $\rho(A) < r$ (d) $\rho(A) \geq r$

(6) If A is a matrix with 6 columns and $rank(A) = 2$ then nullity of A is ___________.
(a) 6 (b) 0 (c) 2 (d) 4

(7) Every orthogonal matrix is ___________.
(a) Hermitian (b) Nilpotent (c) Unitary (d) none

(8) If a 3×3 matrix A has eigen values 2, 3, 5 then $|A| =$___________.
(a) 6 (b) 10 (c) 30 (d) 15

Que.2 Answer the following (Any Five) 10

(1) Prove that $\sin ix = i \sinh x$.

(2) Find all the roots of $\sinh z = i$.

(3) If $A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 4 & 0 \end{bmatrix}$,$B = \begin{bmatrix} 1 & -3 \\ 2 & 0 \\ 6 & 5 \end{bmatrix}$ then find BA .

(4) Show that the function $f : R \to R$ defined by $f(x) = 3x^3 + 5$, $x \in R$ is a bijection .

(5) Determine the values of α, β, γ for which $A = \begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal matrix .

(6) Show that $A = \dfrac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary matrix .

(7) Prove that the characteristic roots of a Hermitian matrix are all real .

(8) What condition must b_1 , b_2 , b_3 satisfy in order for
$x_1 + 2x_2 + 3x_3 = b_1$, $2x_1 + 5x_2 + 3x_3 = b_2$, $x_1 + 8x_3 = b_3$ be consistent ?

Que.3 (a) Prove that $(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\left(\dfrac{\theta}{2}\right)\cos\left(\dfrac{n\theta}{2}\right)$. 4

(b) If $\tan(\theta + i\phi) = e^{i\alpha}$ then prove that $\theta = \left(n+\dfrac{1}{2}\right)\dfrac{\pi}{2}$ and $\phi = \dfrac{1}{2}\log\ \tan\left(\dfrac{\pi}{4}+\dfrac{\alpha}{2}\right)$. 4

OR

Que.3 (c) State and prove De-Moivres theorem . 4

(d) Find all the values of $\left(\frac{1}{2}+\frac{\sqrt{3}\,i}{2}\right)^{3/4}$ .Also prove that the continued product of these values is 1. 4

Que.4 (a) Let X, Y, Z be any non empty sets and let f, g be one one mappings of X onto Y and Y onto Z respectively so that f and g are both invertible.Then prove that gof is also invertible and $(gof)^{-1} = f^{-1}og^{-1}$. 4

(b) Prove that every square matrix can be expressed in one and only one way as the sum of a symmetric and skew-symmetric matrix. 4

OR

Que.4 (c) Let A, B, C, D be sets .Suppose R is a relation from A to B , S is a relation from B to C and T is a relation from C to D .Then show that $(RoS)oT = Ro(SoT)$. 3

(d) If $A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ then prove that $(A_\alpha)^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$,where n is any positive integer. Also prove that A_α and A_β commute and $A_\alpha A_\beta = A_{\alpha+\beta}$. 5

Que.5 (a) Obtain the reduced row echelon form of the matrix $A = \begin{bmatrix} 1 & 3 & 2 & 2 \\ 1 & 2 & 1 & 3 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 8 \end{bmatrix}$ and hence find the rank of the matrix A. 4

(b) Find the inverse of $A = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & \frac{-2}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{10} \\ \frac{1}{5} & \frac{-4}{5} & \frac{1}{10} \end{bmatrix}$ (by Gauss-Jordan Method) 4

OR

Que.5 (c) Reduce $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ to its normal form . 5

(d) Show that $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ is nilpotent matrix of order 3. 3

Que.6 (a) Solve the system $2x+y+z=0$, $3x+2y+3z=18$, $x+4y+9z=16$ (by Gauss Elimination Method) 5

(b) State and prove Cayley-Hamilton theorem. 4

OR

Que.6 (c) Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and verify that it is satisfied by A and hence obtain A^{-1}. 5

(d) Find the characteristic roots and any one characteristic vector of $\begin{bmatrix} -4 & 8 & -12 \\ 6 & -6 & 12 \\ 6 & -8 & 14 \end{bmatrix}$. 3

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