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V.P.& R.P.T.P.Science College.Vallabh Vidyanagar.

Internal Test

B.Sc. Semester III

US03CMTH01 (Advanced Calculus)

5/10/2013 , Saturday 1.00 p.m. to 2.30 p.m.

Maximum Marks: 30

Que.1 Fill in the blanks. 6

(1) $\int_0^2 \int_0^y dxdy =$ .....

(a) 1 (b) 1/2 (c) 0 (d) 2

(2) In double integral , Total mass M of density 1 over region $0 \leq x \leq 2$; $0 \leq y \leq 1$ is .....

(a) 1 (b) 2 (c) 0 (d) 4

(3) In usual notation we say that, $\iint_R \nabla^2 w \, dxdy =$ .....

(a) $\int_C \frac{\partial^2 w}{\partial n^2} \, ds$ (b) $\int_C \frac{dw}{dn} \, ds$ (c) $\frac{1}{2}\int_C \frac{\partial w}{\partial n} \, ds$ (d) 0

(4) If $\bar{v} = y\bar{i} + 4x\bar{j}$ then $\iint_R \nabla \cdot \bar{v} \; dxdy =$ .....

(a) 0 (b) 2 (c) -1 (d) 1

(5) Parametric form of the plane $y = x$ is $\bar{r} =$.....

(a) $u\bar{i} + v\bar{j} + u\bar{k}$ (b) $u\bar{i} + u\bar{j} + v\bar{k}$ (c) $v\bar{i} + u\bar{j} + v\bar{k}$ (d) $u\bar{i} + \bar{j} + v\bar{k}$

(6) Area of a surface $\bar{r}(u, v)$ is A = ...

(a) $\iint_R \sqrt{EG - F^2} \, dxdy$ (b) $\iint_R \sqrt{EG + F^2} \, dxdy$ (c) $\iint_R \sqrt{EG - F^2} \, dudv$

(d) $\iint_R \sqrt{EG - F} \, dudv$

Que.2 Answer the following (Any three) 6

(1) Evaluate $\int_C 3(x^2 + y^2)ds$, where

C : Over the path y = x from (0,0) to (1,1) (counterclockwise direction) .

(2) Find area of the region bounded by $y = x^2$ and $y = 2x + 3$.

(3) Prove that area of plane region in cartesian form is given by $A = \frac{1}{2}\int_C [xdy - ydx]$.

(4) Evaluate the line integral $\int_{(2,0,0)}^{(1,2,3)} [xdx + ydy + zdz]$ on any path.

(5) Represent the surface $z^2 = x^2 + y^2$ in parametric form .

(6) By using divergence theorem , evaluate $\iint_S [x^2 dydz + y^2 dzdx + z^2 dxdy]$,

where S: The surface of cube $0 \leq x$, y , $z \leq 1$.

Que.3 Find volume of the region bounded by the tetrahedral cut from the first octant by the plane $3x + 4y + 2z = 12$. 6

OR

Que.3 Find the centroid of density 1 in the plane area bounded by $y = 2x - x^2$ and $y = 3x^2 - 6x$. 6

Que.4 State and prove Green's theorem for plane . 6

OR

Que.4 Verify the result $\iint_R \nabla \cdot \overline{V}\, dxdy = \int_C \overline{V} \cdot \overline{n}\, ds$

for $\overline{V} = 7x\bar{i} - 3y\bar{j}$, C : the circle $x^2 + y^2 = 4$. 6

Que.5 State and prove divergence theorem of Gauss. 6

OR

Que.5 Find moment of inertia of surface S of density 1 about z-axis , where

$S : \bar{r} = (a + b\cos v)(\cos u\bar{i} + \sin u\bar{j}) + b\sin v\bar{k}$, $a > b > 0$, $0 \leq u, v \leq 2\pi$. 6

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