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V.P.& R.P.T.P.Science College,Vallabh Vidyanagar.
Internal Test
B.Sc. Semester III
US03CMTH01 (Advanced Calculus)

Date. 5/10/2015 ; Monday 3.00 p.m. to 4.30 p.m. Maximum Marks: 25

Que.1 Fill in the blanks. 3

(1) For $x+y=u$, $x-2y=v$, jacobian $|J|=$ ....
(a) $\frac{1}{3}$ (b) $\frac{-1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

(2) If $\bar{v}=7x\bar{i}-3y\bar{j}$ then $\iint_R (\nabla\times\bar{v})\cdot\bar{k}\ dxdy=$ ....
(a) 1 (b) 2 (c) -1 (d) 0

(3) If $\bar{r}=u\cos v\ \bar{i}+u\sin v\ \bar{j}+u\ \bar{k}$ then $EG-F^2=$ ....
(a) 2 (b) 0 (c) $2u^2$ (d) u^2

Que.2 Answer the following (Any Two) 4

(1) Evaluate $\iint_R e^{-x^2-y^2}\ dxdy$; where $R: x^2+y^2=1$.

(2) Show that in $\int_{(2,0,0)}^{(-1,2,\pi)} [ye^{xy}\cos z dx + xe^{xy}\cos z dy - e^{xy}\sin z dz]$,the form under integral sign is exact and hence evaluate it .

(3) State and prove first fundamental form of a surface in cartesian form .

Que.3 (a) Transform $\iint_R (x^2+y^2)\,dxdy$ in uv-plane by taking $x+y=u$, $x-y=v$. Then evaluate it , where R : Parallelogram with vertices $(0,0)$, $(1,1)$, $(2,0)$, $(1,-1)$. 4

(b) Find area of the region bounded by parabola $y^2=4-x$ *and* $y^2=4-4x$. 2

OR

Que.3 (a) Find volume of the region bounded by the first octant section cut from the region inside the cylinder $x^2+z^2=1$ and by the plane $y=0, z=0, x=y$. 5

(b) Evaluate $\int_0^{\pi/2}\int_0^1 x^2y^2\,dydx.$ 1

Que.4 (a) State and prove Green's theorem for plane . 4

(b) Evaluate $\int_C \frac{\partial w}{\partial n}\,ds$, where $w=2x^2+y^2$ and C : the boundary of the region bounded by $y=x^2$ and $y=x+2$. 2

OR

Que.4 (a) Change the order of integration in $\int_0^{2a}\int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} f(x,y)\,dydx.$ 3

(b) Evaluate $\int_C [(3x^2-8y^2)dx+(4y-6xy)dy]$ by using Green's theorem , where C : the boundary of region bounded by $y=\sqrt{x}$ and $y=x^2$. 3

Que.5 (a) State and prove divergence theorem of Gauss. 5

(b) By using divergence theorem , evaluate $\iint_S [x^3dydz+x^2ydzdx+x^2zdxdy]$, where S : closed surface bounded by the plane z = 0 , z = b , $x^2+y^2=a^2$. 1

OR

Que.5 (a) Find area of the surface $z^2=x^2+y^2$, where $0\le z\le 1$. 3

(b) Evaluate $\iint_S f(x,y,z)\,dA$, where $f(x,y,z)=xy$ and S : $z=xy$, $0\le x$, $y\le 1$. 3

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