V.P. & R.P.T.P. Science College, V.V. Nagar

Internal Test: 2016-17

Subject: Mathematics

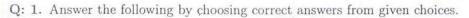
US04CMTH02

Max. Marks: 25

Differential Equations

Date: 11/03/2017

Timing: 3.00 pm - 04.30 pm



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- [1] Integral curve of $\frac{dx}{x}=\frac{dy}{y}=\frac{dz}{z}$ is given by [A] $x=c_1y$; $y=c_2z$ [B] $yz=c_1x$; $xz=c_2y$ [C] yz+xz+xy=0 [D] x+y+z=0



- [2] A Paffian differential equation in two variables
 - [A] has no solution
 - can have solution if certain conditions are satisfied
 - [C] always possesses a solution
 - [D]none of these
- [3] A surface orthogonal to given system of surfaces cuts them at an angle measuring
 - $[A] \pi$

- $[B] \frac{\pi}{2}$
- [C] $\frac{\pi}{3}$

 $[D] \frac{\pi}{6}$

Answer any TWO of the following.

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- [1] Find the integral curves of the equations $\frac{dx}{1+x} = \frac{dy}{1+y} = \frac{dz}{z}$
- [2] Determine whether the equation ydx + xdy = 5zdz is integrable or not.
- [3] Obtain a differential equation of the form $\frac{dx}{P}=\frac{dy}{Q}=\frac{dy}{R}$ whose integral curves generate surfaces orthogonal to the surfaces $x^2-2y^2-4z^2=c$

Q: 3 [A] Solve :
$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$$

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[B] Find the integral curves of the equations

$$\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}$$

3

OR

Q: 3 [A] Solve:
$$\frac{dx}{z} = \frac{dy}{-z} = \frac{dz}{z^2 + (x+y)^2}$$

3

[B] Find the orthogonal trajectories on the cone $x^2+y^2=z^2\tan^2\alpha$ of its intersection with the family of planes parallel to z = 0

3

Q: 4 [A] If X is a vector such that X.curl X = 0 and μ is an arbitrary function of x, yand z then prove that $(\mu X).curl(\mu X) = 0$

3

[B] Solve: $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x+y)z$ 3

OR

Q: 4 [A] Prove that the general solution of the linear differential equation pP+qQ=Ris F(u,v) = 0, where F(u,v) = 0 is an arbitrary function of $u(x,y,z) = c_1$ and $v(x,y,z) = c_2$ which form a solution of the equation $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

3

[B] Determine whether the Pfaffian differential equation

$$(y+z)dx + (z+x)dy + (x+y)dz = 0$$

is integrable or not. Find its solution if it is integrable

3

Find the integral surface of the linear partial differential equation Q: 5. 2y(z-3)p+(2x-z)q=y(2x-3) passing through the circle $z=0,x^2+y^2=2x$

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OR

Verify $z^2 + \mu = 2(1 + \lambda^{-1})(x + \lambda y)$ is a complete integral of partial differential equation $z = \frac{1}{p} + \frac{1}{q}$. Also show that the complete integral is the envelope Q: 5. of the one-parameter subsystem obtained by taking $b = -\frac{a}{\lambda} - \frac{\mu}{1+\lambda}$ in the solution $z = \sqrt{2x+a} + \sqrt{2y+b}$ of the differential equation.

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