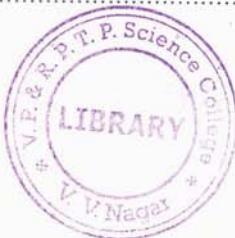


Que.1 Fill in the blanks.

3

- (1) For $x + y = u$, $x - 2y = v$, jacobian $J = \dots$
 (a) $\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$
- (2) Area of plane region $r = a(1 + \cos \theta)$ is $A = \dots$
 (a) $\frac{3\pi a^2}{2}$ (b) $\frac{3\pi a}{2}$ (c) $\frac{\pi a^2}{2}$ (d) $\frac{3a^2}{2}$
- (3) If $\bar{r} = u \bar{i} + v \bar{j} + uv \bar{k}$ then $EG - F^2 = \dots$
 (a) $1 + v^2$ (b) uv (c) $1 + v^2 + u^2$ (d) $1 + u^2$



Que.2 Answer the following (Any Two)

4

- (1) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2) dy dx$.
 (2) Evaluate the line integral $\int_{(0,0,1)}^{(1,\pi/4,2)} [2xyz^2 dx + (x^2 z^2 + z \cos yz) dy + (2x^2 yz + y \cos yz) dz]$ on any path .
- (3) Prove first fundamental form of a surface in cartesian form .
- Que.3 (a) Transform $\iint_R (x-y)^2 \sin^2(x+y) dx dy$ in uv-plane by taking $x-y=u, x+y=v$.
 Then evaluate it, where R : Parallelogram with vertices $(\pi, 0), (2\pi, \pi), (\pi, 2\pi), (0, \pi)$.
- (b) Find area of the region bounded by $y = x^2$ and $y = 2x + 3$.
- OR
- Que.3 (a) Find the centroid of density 1 in the plane area bounded by $y = 2x - x^2$ and $y = 3x^2 - 6x$.
 (b) Evaluate $\int_C (x^2 + y^2) ds$, where
 C : Over the path $y = -x$ from $(-1,1)$ to $(1,-1)$ (counterclockwise direction) .

Que.4 (a) State and prove Green's theorem for plane .

3

(b) Verify the result $\iint_R (\nabla \times \bar{V}) \cdot \bar{k} dx dy = \int_C \bar{V} \cdot \bar{u} ds$ for $\bar{V} = y\bar{i} + 4x\bar{j}$ C : the boundary of triangle with vertices $(0,0), (2,0), (2,1)$

OR

Que.4 (a) Change the order of integration in $\int_0^{2a} \int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} f(x,y) dy dx$.

4

(b) Prove that area of plane region in polar form are given by $A = \frac{1}{2} \int_C r^2 d\theta$.

2

Que.5 (a) State and prove divergence theorem of Gauss .

4

(b) By using divergence theorem , evaluate $\iint_S [x^3 dy dz + x^2 y dz dx + x^2 z dx dy]$,where S: closed surface bounded by the plane $z = 0$, $z = b$, $x^2 + y^2 = a^2$.

2

OR

Que.5 (a) Evaluate $\iint_S f(x,y,z) dA$, where $f(x,y,z) = xy$ and S : $z = xy$, $0 \leq x$, $y \leq 1$.(b) In usual notation find $\sqrt{EG - F^2}$ for surface $\bar{r} = (a + b \cos v)(\cos u\bar{i} + \sin u\bar{j}) + b \sin v\bar{k}$.

2

