V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test : 2013-14

T.Y.B.Sc. : Semester - V (CBCS)

Subject : Mathematics   US05CMTH01 Real Analysis-I   Max. Marks : 30

Date: 30/09/2013   Timing: 3.30 pm - 5.00pm

Instructions : (1) This question paper contains FIVE QUESTIONS
(2) The figures to the right side indicate full marks of the corresponding question/s
(3) The symbols used in the paper have their usual meaning, unless specified

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Q: 1. Answer the following by choosing correct answers from given choices. **6**

[1] If S is a non-empty and bounded above subset of R then there exists

[A] supremum of S in Q   [B] infimum of S in Q

[C] supremim of S in R   [D] infimum of S in R

[2] The infimum of the set $-1, 1, -1\frac{1}{2}, 1\frac{1}{2}, -1\frac{1}{3}, 1\frac{1}{3}, \ldots$

[A] -1   [B] 0

[C] $-1\frac{1}{2}$ [D] $\frac{1}{2}$

[3] For $S = (1,4) \cup \{5,6\}$, 4 is

[A] a limit point of S
[B] an interior point of S
[C] interior point as well as limit point of S
[D] none

[4] In $(0, \frac{\pi}{2})$ function C(x) is

[A] strictly increasing   [B] strictly decreasing

[C] stationary   [D] none

[5] $\lim_{x \to 0-} e^{\frac{1}{x}} =$

[A] 0   [B] 1

[C] ∞ [D] $-\infty$

[6] If $\lim_{x \to a-} f(x)$ and $\lim_{x \to a+} f(x)$ both do not exist then then f is said to have a discontinuity of

[A] removable type [B] first type

[C] second type [D] first type from right

Q: 2. Answer any THREE of the following. 6

[1] Define Complete Ordered Field.

[2] Find the g.l.b and l.u.b. of $\left\{1 + \frac{(-1)^n}{2} \; / n \in N\right\}$ if they exist.

[3] Determine whether the interior of the set $[2, 8] \cup (9, 10) \cap N$ is open or not.

[4] Find the set of all the interior points of $\{1, 2, 3, e, \pi\}$

[5] Prove that the function deined on $\Re$ by

$$f(x) = \begin{cases} -1\ ; & \text{when x is irrational} \\ 1\ ; & \text{when x is rational} \end{cases}$$

is not continuous at every point

[6] Examine the function

$$f(x) = \begin{cases} x^2 + 2x \text{ when } & x \neq 3 \\ 15, \quad \text{when} & x = 3 \end{cases}$$

for continuity at $x = 3$

Q: 3. Prove that the set of rational numbers is not order complete. 6

OR

Q: 3 [A] State and prove the Archimedean property of R and deduce that for any real number c there exists a positive integer n such that $n > c$. 3

[B] In usual notations prove that $L(ab) = L(a).L(b)$. 3

Q: 4. Show that every bounded infinite set has the smallest and the greatest limit point. 6

OR

Q: 4 [A] Show that the interior of a set is an open set. 3

[B] Show that every open set is a union of open intervals. 3

Q: 5. If a function f is continuous on $[a, b]$ and $f(a)$ and $f(b)$ are of opposite signs, then there exists at least one point $\alpha \in (a, b)$ such that $f(\alpha) = 0$. 6

OR

Q: 5. Show that a function $f : [a, b] \to \Re$ is continuous at point c of [a,b] iff

$$\lim_{n \to \infty} c_n = c \Longrightarrow \lim_{n \to \infty} f(c_n) = f(c)$$

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