V.P. & R.P.T.P. Science College,V.V.Nagar

Internal Test : 2013-14

T.Y.B.Sc. : Semester - V (CBCS)

Subject : Mathematics — US05CMTH02 Real Analysis-II — Max. Marks : 30

Date: 01/10/2013 — Timing: 3.30 pm - 5.00pm

Instructions : (1) This question paper contains FIVE QUESTIONS
(2) The figures to the right side indicate full marks of the corresponding question/s
(3) The symbols used in the paper have their usual meaning, unless specified

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Q: 1. Answer the following by choosing correct answers from given choices. **6**

[1] The sequence $\{S_n\}_{n=1}^{\infty}$, where $S_n = (-1)^n \left(1 + \frac{1}{n}\right)$

[A] is convergent [B] oscillates finitely

[C] oscillates infinitely [D] is divergent

[2] Every convergent sequence is

[A] oscillating [B] bounded

[C] unbounded [D] none

[3] A positive term series $\sum\limits_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if and only if

[A] $p < 1$ [B] $p > 1$

[C] $p \leqslant 1$ [D] $p \geqslant 1$

[4] The positive term series $\sum\limits_{n=1}^{\infty} u_n$ is convergent if

[A] $\sum\limits_{n=1}^{\infty} \frac{u_{n+1}}{u_n} = 1$ [B] $\sum\limits_{n=1}^{\infty} \frac{u_{n+1}}{u_n} < 1$

[C] $\sum\limits_{n=1}^{\infty} \frac{u_{n+1}}{u_n} > 1$ [D] none

[5] If $f(x, y) = x^3y^3 - 3x^2y^2$ then $f_y(0, 1) =$

[A] 0 [B] 1

[C] 2 [D] 3

[6] $\lim\limits_{(x,y)\to(4,\pi)} x^2 \sin\frac{y}{x} =$

[A] 8 [B] $-8\sqrt{2}$

[C] $8\sqrt{2}$ [D] 0

Q: 2. Answer any THREE of the following. 6

[1] For any number x show that $\lim\limits_{n\to\infty} \frac{x^n}{n!} = 0$

[2] Using the definition of limit show that $\lim\limits_{x\to -2} 3x+7 = 1$

[3] If $\sum\limits_{n=1}^{\infty} u_n = u$ and $\sum\limits_{n=1}^{\infty} v_n = v$ then prove that $\sum\limits_{n=1}^{\infty}(u_n+v_n) = u+v$

[4] Test for convergence of the series whose general term is $\dfrac{2n+1}{n}$

[5] If

$$f(x,y) = \begin{cases} x^2+2y\ ; & \text{when } (x,y) \neq (1,2) \\ 0\ ; & \text{when } (x,y) \neq (1,2) \end{cases}$$

then show that f is discontinuous at $(1,2)$

[6] Evaluate : $\lim\limits_{(x,y)\to(0,0)} \frac{\sin^{-1}(xy-2)}{\tan^{-1}(3xy-6)}$

Q: 3. Define Convergent Sequence and show that every convergent sequence is bounded and has a unique limit. 6

OR

Q: 3. Show that the sequence $\{r^n\}$ converges iff $-1 < r \leq 1$. 6

Q: 4 [A] Prove that a positive term series is convergent it and only if the sequence of its partial sums is bounded above. 3

[B] Investigate the behaviour of the series whose n^{th} term is $\sin\dfrac{1}{n}$ 3

OR

Q: 4. State and prove the D'-Alembert's Ratio test 6

Q: 5. If V is a function of two variables x and y and $x = r\cos\theta$, $y = r\sin\theta$ then prove that

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = \frac{\partial^2 V}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{1}{r}\frac{\partial V}{\partial r}$$

6

OR

Q: 5. Show that $f(xy, z-2x) = 0$ satisfies, under suitable conditions, the equation $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 2x$. What are these conditions? 6