



V.P. & R.P.T.P. Science College, V.V. Nagar

Internal Test: 2013-14

T.Y.B.Sc. : Semester - V (CBCS)

Subject : Mathematics

US05CMTH03  
Metric Spaces

Max. Marks : 30

Date: 03/10/2013

Timing: 3.30 pm - 5.00pm

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- Instructions : (1) This question paper contains FIVE QUESTIONS  
(2) The figures to the right side indicate full marks of the corresponding question/s  
(3) The symbols used in the paper have their usual meaning, unless specified
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Q: 1. Answer the following by choosing correct answers from given choices.

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[ 1 ] Every function defined on  $R_d$  is

- [A] continuous
- [B] discontinuous
- [C] continuous only at rational points
- [D] continuous only at irrational points

[ 2 ] Every Cauchy sequence is

- [A] convergent
- [B] is not always convergent
- [C] divergent
- [D] none

[ 3 ] In the metric space  $M = [0, 1]$  with usual metric,  $B[\frac{1}{4}, 1] =$

- [A]  $[0, 1]$
- [B]  $[\frac{1}{4}, 1]$
- [C]  $[0, \frac{1}{4}]$
- [D]  $(0, 1)$

[ 4 ] The set  $\{1, 2, 3, 4\}$  is

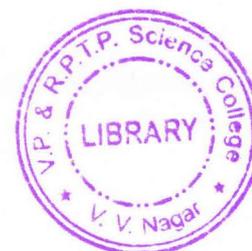
- [A] open in  $R_d$  but closed in  $R^1$
- [B] open in  $R_d$  and  $R^1$  both
- [C] closed in  $R_d$  but open in  $R^1$
- [D] none

[ 5 ] The range of a continuous function  $f$  defined on  $[1, 2]$  is

- [A] unbounded
- [B] compact
- [C] not compact
- [D] none



- [ 6] Every finite subset of a metric space is
- [A] unbounded [B] compact
- [C] not compact [D] none



Q: 2. Answer any THREE of the following. 6

- [ 1] Define : (i) Metric Space (ii) Cluster Point
- [ 2] Let  $(M, d)$  be a metric space and let  $d^*(x, y) = \min\{1, d(x, y)\}$ . Then prove that  $d^*$  is a metric on  $M$
- [ 3] Define (i) Connected set (ii) Limit point
- [ 4] Prove that every constant function  $f : R^1 \rightarrow R^1$  is continuous
- [ 5] Prove that  $g(x) = \sqrt{x}$ ,  $x \in [0, \infty)$  is continuous on  $[0, \infty)$
- [ 6] Show that the range of a continuous function, on a compact metric space, is bounded.

Q: 3. Prove that a real valued function  $f$  is continuous at  $a \in R$  iff

$$\lim_{n \rightarrow \infty} x_n = a \implies \lim_{n \rightarrow \infty} f(x_n) = f(a)$$

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OR

Q: 3. Define Cauchy Sequence. Also prove that if  $\{s_n\}_{n=1}^{\infty}$  is a convergent sequence of points in a metric space  $(M, \rho)$  then  $\{s_n\}_{n=1}^{\infty}$  is Cauchy. Is the converse true? Justify.

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Q: 4 [A] Prove that if  $F_1$  and  $F_2$  are closed subsets of a metric space  $M$  then  $F_1 \cup F_2$  is closed in  $M$

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[ B] If  $E$  is any subset of a metric space  $M$  then prove that  $\overline{E}$  is closed in  $M$

3

OR

Q: 4. Define a Connected Set. Also Prove that a subset  $A$  of  $R^1$  is connected iff whenever  $a \in A$ ,  $b \in A$  with  $a < b$ , then  $c \in A$  for every  $c$  such that  $a < c < b$ .

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Q: 5 [A] Let  $f$  be a real valued continuous function on  $[a, b]$ . Then prove that  $f$  is bounded.

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[ B] Give an example of a function which is one-one, onto, continuous but its inverse is not continuous.

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OR

Q: 5. Let  $(M_1, \rho_1)$  be a metric space and let  $A$  be a dense subset of  $M_1$ . If  $f$  is a uniformly continuous function from  $(A, \rho_1)$  into a complete metric space  $(M_2, \rho_2)$  then prove that  $f$  can be extended to a uniformly continuous function  $F$  from  $M_1$  into  $M_2$ .

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