V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2013-14

T.Y.B.Sc. : Semester - V (CBCS)

Subject : Mathematics | US05CMTH03 Metric Spaces | Max. Marks : 30

Date: 03/10/2013 | Timing: 3.30 pm - 5.00pm

Instructions : (1) This question paper contains FIVE QUESTIONS
(2) The figures to the right side indicate full marks of the corresponding question/s
(3) The symbols used in the paper have their usual meaning, unless specified

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Q: 1. Answer the following by choosing correct answers from given choices. **6**

[1] Every function defined on R_d is

- [A] continuous
- [B] discontinuous
- [C] continuous only at rational points
- [D] continuous only at irrational points

[2] Every Cauchy sequence is

[A] convergent [B] is not always convergent

[C] divergent [D] none

[3] In the metric space $M = [0,1]$ with usual metric , $B[\frac{1}{4}, 1] =$

[A] $[0,1]$ [B] $[\frac{1}{4}, 1]$

[C] $[0, \frac{1}{4}]$ [D] $(0,1)$

[4] The set $\{1,2,3,4\}$ is

[A] open in R_d but closed in R^1 [B] open in R_d and R^1 both

[C] closed in R_d but open in R^1 [D] none

[5] The range of a continuous function f defined on $[1,2]$ is

[A] unbounded [B] compact

[C] not compact [D] none

[6] Every finite subset of a metric space is

[A] unbounded   [B] compact

[C] not compact   [D] none

Q: 2. Answer any THREE of the following. **6**

[1] Define : (i) Metric Space (ii) Cluster Point

[2] Let (M, d) be a metric space and let $d^*(x, y) = min\{1, d(x, y)\}$. Then prove that d^* is a metric on M

[3] Define (i) Connected set (ii) Limit point

[4] Prove that every constant function $f : R^1 \longrightarrow R^1$ is continuous

[5] Prove that $g(x) = \sqrt{x},\ x \in [0, \infty)$ is continuous on $[0, \infty)$

[6] Show that the range of a continuous function, on a compact metric space, is bounded.

Q: 3. Prove that a real valued function f is continuous at $a \in R$ *iff*

$$\lim_{n\to\infty} x_n = a \implies \lim_{n\to\infty} f(x_n) = f(a)$$

6

OR

Q: 3. Define Cauchy Sequence. Also prove that if $\{s_n\}_{n=1}^{\infty}$ is a convergent sequence of points in a metric space (M, ρ) then $\{s_n\}_{n=1}^{\infty}$ is Cauchy. Is the converse true? Justify. **6**

Q: 4 [A] Prove that if F_1 and F_2 are closed subsets of a metric space M then $F_1 \cup F_2$ is closed in M **3**

[B] If E is any subset of a metric space M then prove that $\overline{E}$ is closed in M **3**

OR

Q: 4. Define a Connected Set. Also Prove that a subset A of R^1 is connected iff whenever $a \in A,\ b \in A$ with $a < b$, then $c \in A$ for every c such that $a < c < b$.

6

Q: 5 [A] Let f be a real valued continuous function on $[a, b]$. Then prove that f is bounded.

3

[B] Give an example of a function which is one-one, onto, continuous but its inverse is not continuous. **3**

OR

Q: 5. Let (M_1, ρ_1) be a metric space and let A be a dense subset of M_1. If f is a uniformly continuous function from (A, ρ_1) into a complete metric space (M_2, ρ_2) then prove that f can be extended to a uniformly continuous function F from M_1 into M_2 . **6**