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V.P.& R.P.T.P.Science College.Vallabh Vidyanagar.
Internal Test
B.Sc. Semester V
US05CMTH04 (Abstract Algebra)
4/10/2013 , Friday 3.30 p.m. to 5.00 p.m.

Maximum Marks: 30

Que.1 Fill in the blanks. 6

(1) Multiplicative inverse of 6 in Z_7^* is ....
(a) 3 (b) 6 (c) 2 (d) 1
(2) In Klein 4-group $G = \{e, a, b, c\}$, abc = ...
(a) c (b) e (c) b (d) a
(3) .... is generator of group Z_5^*.
(a) $\bar{0}$ (b) $\bar{1}$ (c) $\bar{4}$ (d) $\bar{2}$
(4) $O(\bar{5})$ in Z_6 is ....
(a) 6 (b) 3 (c) 4 (d) 2
(5) Every cyclic group of order 4 is isomorphic to ....
(a) Klein 4-group (b) Z (c) N (d) Z_4
(6) If H is any normal subgroup of G then ....
(a) $Hx = Hy$ (b) $Hx = xH$ (c) $Hx = H$ (d) $xH = yH$

Que.2 Answer the following (Any three) 6

(1) In group G ,prove that every element of G has unique inverse.
(2) Prove or disprove:Union of two subgroups of group is also a subgroup.
(3) Find all generators of group $\{\pm 1, \pm i\}$,if possible.
(4) Let H be any subgroup of group G.Then prove that $aH = H \Leftrightarrow a \in H$.
(5) Prove that isomorphic image of abelian group is also abelian.
(6) Prove that $\theta : Z \to Z$ defined by $\theta(n) = -n$ is an automorphism of Z.

Que.3 Prove that $(G, \cdot)$ is a non-commutative group,where G is set of all 2×2 non singular matrices . 6

OR

Que.3 Let H and K be subgroups of group G.Then prove that HK is subgroup of G iff $HK = KH$. 6

Que.4 State and prove Lagrange's theorem , Euler's theorem and Fermat's theorem . 6

OR

Que.4 Let G be a cyclic group and H , a subgroup of G.Then prove that H is cyclic. 6

Que.5 State and prove Third isomorphism theorem. 6

OR

Que.5 Let $G = (a)$ be a finite cyclic group of order n.Then prove that the mapping $\theta : G \to G$ defined by $\theta(a) = a^m$ is an automorphism of G iff m is relatively prime to n. 6

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