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V.P.& R.P.T.P.Science College.Vallabh Vidyanagar.
Internal Test
B.Sc. Semester V
US05CMTH05 (Number Theory)
5/10/2013 , Saturday 3.30 p.m. to 5.00 p.m.

Maximum Marks: 30

Que.1 Fill in the blanks. 6

(1) $(4676, 366) =$ ....
(a) 2 (b) 6 (c) 4 (d) 1
(2) $[12, 30] =$ ....
(a) 6 (b) 60 (c) 360 (d) 30
(3) If a is square number then S(a) is ....
(a) even (b) odd (c) prime (d) 0
(4) .... is a Mersenne number .
(a) 16 (b) 6 (c) 15 (d) 31
(5) $ax + by = c$ has integer solution if and only if ....
(a) (a , b) = a (b) (a , b) = b (c) $(a,b)/c$ (d) $c/(a,b)$.
(6) 765432 is divided by ....
(a) 5 (b) 3 (c) 11 (d) 13

Que.2 Answer the following (Any three) 6

(1) Find gcd of two numbers by using Euclidean algorithm .
(2) Prove that $(a+b)[a,b] = b[a, a+b]$,$\forall\ a,b > 0$.
(3) If a is not square number but odd integer then prove that S(a) is even integer .
(4) If $m = qn + r$ then prove that $(u_m, u_n) = (u_n, u_r)$.
(5) If $ca \equiv cb (mod n)$ and (c,n)=1 then prove that $a \equiv b (mod n)$.
(6) If $a_1 \equiv b_1 (mod n)$,then prove that ${a_1}^m \equiv {b_1}^m (mod n)$, $\forall\ m \in \mathbb{N}$, by using mathematical induction method.

Que.3 Let g be a positive integer greater than 1 then prove that every positive integer a can be written uniquely in the form
$a = c_n\, g^n + c_{n-1}\, g^{n-1} + ... + c_1\, g + c_0$, where $n \geq 0$, $c_i \in \mathbb{Z}$, $0 \leq c_i < g$, $c_n \neq 0$. 6

OR

Que.3 State and prove unique factorization theorem for positive integers. 6

Que.4 Prove that every prime factor of $F_n (n > 2)$ is of the form $2^{n+2}\, t + 1$, for some integer t. 6

OR

Que.4 Prove that odd prime factor of M_p $(p > 2)$ has the form $2pt + 1$, for some integer t. 6

Que.5 Prove that the integer solution of $x^2 + 2y^2 = z^2$, $(x, y) = 1$ can be expressed as
$x = \pm(a^2 - 2b^2)$, $y = 2ab$, $z = a^2 + 2b^2$. 6

OR

Que.5 Solve the equation $19x + 20y = 1909$. 6

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