V.P. & R.P.T.P. Science College,V.V.Nagar

Internal Test : 2013-14
T.Y.B.Sc. : Semester - 6 (CBCS)

Subject : Mathematics | US06CMTH01 Real Analysis - 3 | Max. Marks : 30

Date: 10/03/2014 | Timing: 3.30 pm - 5.00pm

Instructions : (1) This question paper contains FOUR QUESTIONS
(2) The figures to the right side indicate full marks of the corresponding question/s
(3) The symbols used in the paper have their usual meaning, unless specified

Q: 1. Answer any THREE of the following. **6**

[**1**] Explain the geometric meaning of Lagrange's Mean Value theorem

[**2**] State Rolle's theorem

[**3**] Can two partitions of $[a, b]$ be disjoint sets? Justify.

[**4**] Find the mesh of the partition $\{2, 3, 5, 7, 10, 11, 13\}$ of $[2, 13]$

[**5**] Is $f(x) = [x]$ an integrable function over $[0, 5]$? Justify.

[**6**] If a function f has five points of discontinuity in $[1, 4]$ then can it be integrable over $[1, 4]$? Justify.

Q: 2 [A] A twice differentiable function f is such that $f(a) = f(b) = 0$ and $f(c) > 0$ for $a < c < b$. Prove that there is al least one value ξ between a and b for which $f''(\xi) < 0$. **4**

[**B**] Show that $\dfrac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \cot\theta$, for some θ where $0 < \alpha < \theta < \beta < \dfrac{\pi}{2}$ **4**

OR

Q: 2 [A] State and prove Maclaurin's theorem. **4**

[**B**] Examine the validity of the hypothesis and the conclusion of Lagrange's Mean Value theorem for the function $f(x) = 2x^2 - 7x + 10$ on $[2, 5]$ **4**

Q: 3 [A] If a refinement P^* of a partition P contains p points more than the points in P and $|f| \leqslant k, \forall x \in [a, b]$ then prove that $L(P, f) \leqslant L(P^*, f) \leqslant L(P, f) + 2pk\mu$ **4**

[B] If f is a bounded and integrable function on $[a, b]$ and k is a number such that $|f| \leqslant k,\ \forall x \in [a, b]$ then prove that

$$\left| \int_a^b f.dx \right| \leqslant k|b - a|$$

4

OR

Q: 3 [A] If f is bounded on $[a, b]$ then for any two partitions P_1 and P_2 of $[a, b]$, prove that $L(P_1, f) < U(P_2, f)$ 4

[B] Show that a constant function k is integrable and evaluate $\int_a^b k.dx$ 4

Q: 4. Define Riemann Sum. Also show that a function f is integrable over $[a, b]$ iff for $\epsilon > 0$, there exists $\delta > 0$ such that if P, P' are any two partitions of $[a, b]$ with mesh less than δ then

$$|S(P, f) - S(P', f)| < \epsilon$$

8

OR

Q: 4 [A] Show that every continuous function is integrable 4

[B] Prove that a bounded function f having a finite number of points of discontinuty on $[a, b]$ is integrable on $[a, b]$. 4