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V.P.& R.P.T.P.Science College.Vallabh Vidyanagar.
Internal Test
B.Sc. Semester VI
US06CMTH02 (Complex Analysis)
Tuesday , 11^{th} March 2014
3.30 p.m. to 5.00 p.m.

Maximum Marks: 30

Que.1 Answer the following (Any three) 6

(1) By using definition, prove that $\frac{d}{dz}(z^{-1}) = -\frac{1}{z^2}$.

(2) Prove that limit of function is unique,if it exist.

(3) Prove that $f(z) = 2x + ixy^2$ is not differentiable at any point .

(4) Verify that $f(z) = e^{ix+y}$ is nowhere analytic .

(5) Prove that $sinz = sinxcoshy + icosxsinhy$.

(6) Find all values of $cosh^{-1}(-1)$.

Que.2 (a) By using definition of limit prove that $\lim_{z\to(1-i)} (x + i(2x + y)) = 1 + i$. 4

(b) Prove that a composition of a continuous function is also continuous. 4

OR

Que.2 (a) Prove that every differentiable function is continuous.Does the converse hold ? Verify it. 6

(b) If $f(z) = \frac{x^3y(y - ix)}{z(x^6 + y^2)}$, $z \neq 0$, $f(0) = 0$. Is $\lim_{z\to 0} f(z)$ exists ? 2

Que.3 (a) Give an example of function such that partial derivatives of its components satisfies the C-R equations at some points but function is not differentiable at that point . Verify it . 6

(b) Check that whether $f(z) = (3x + y) + i(3y - x)$ is entire or not . 2

OR

Que.3 (a) State and prove sufficient conditions for differentiability of f(z). 5

(b) Find a harmonic conjugate v(x,y) for $u(x, y) = 2x - x^3 + 3xy^2$. 3

Que.4 (a) Prove that $cosz_1 - cosz_2 = -2\sin\left(\frac{z_1 + z_2}{2}\right)\sin\left(\frac{z_1 - z_2}{2}\right)$. 3

(b) Solve the equation $e^z = \sqrt{3} - i$ 3

(c) Prove that $\overline{exp(iz)} = exp(i\bar{z})$ if $z = n\pi$, $n \in \mathbb{Z}$. 2

OR

Que.4 (a) Find all values of $cos^{-1}(\sqrt{2})$. 3

(b) Prove that $Log(-1+i) = \frac{1}{2}\, ln2 + 3\frac{\pi}{4}i.$ 3

(c) Prove that $\frac{d}{dz}(tanhz) = sech^2 z.$ 2

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