V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2013-14

T.Y.B.Sc. : Semester - 6 (CBCS)

Subject : Mathematics | US06CMTH03 | Max. Marks : 30

Topology

Date: 12/03/2014 | Timing: 3.30 pm - 5.00pm

Q: 1. Answer any THREE of the following. 6

[1] Show that the sets $\mathbb{R}$ and $\emptyset$ are $\mathcal{U}$-open.

[2] Define : (i) Topology (ii) Open Set

[3] Define : (i) Closure of a set (ii) Interior point

[4] Find the sets of cluster points of $(1, 2]$ in $\mathcal{U}$-topology of $\mathcal{R}$.

[5] Prove that discrete space that has more than one point disconnected

[6] Assuming that connectedness is a topological property prove that $(R, \mathcal{U})$ and (R, ψ) are not homeomorphic

Q: 2 [A] Let J be the set of all integers and $\mathcal{T}$ be a collection of subsets G of J where $G \in \mathcal{T}$ whenever $G = \emptyset$ or $G \neq \emptyset$ and $p, p \pm 2, p \pm 4, ..., p \pm 2n, ...$ belong to G whenever $p \in G$. Prove that $\mathcal{T}$ is a non-trivial topology for J 4

[B] Find three mutually non-comparable topologies of $X = \{a, b, c\}$ 4

OR

Q: 2 [A] Show in two ways that if $a \in \mathbb{R}$ then $\{a\}$ is closed in the usual topology of $\mathbb{R}$ 4

[B] Are closed intervals of $\mathbb{R}$, $\mathcal{U}$-closed? where $\mathcal{U}$ is the usual topology for $\mathbb{R}$ 4

Q: 3 [A] Let $(X, \mathcal{T})$ be a topological space and a A be a subset of X. Then prove that - (i) $Int(A) \subset A$ (ii) $Int(A)$ is a $\mathcal{T}$-open set 4

[B] Find the $\mathcal{U}$-closure of each : (a) $\mathbb{R}$ (b) $\emptyset$ (c) $[0, 1]$ (d) $[0, 1)$ 4

OR

Q: 3 [A] Let $(X, \mathcal{T})$ be a topological space and let A be a subset of X. Then prove that $A^- = A \cup A'$ where A' be the set of all cluster points of A. 4

[B] For any topologies $\mathcal{T}$ and Ψ of $\mathbb{R}$ show that the mapping $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = 2, \forall x \in \mathbb{R}$, is $\mathcal{T}$-Ψ continuous. 4

Q: 4 [A] Prove that if $(X, \mathcal{T})$ is disconnected then there is a nonempty proper subset of X that is both $\mathcal{T}$-open and $\mathcal{T}$-closed. 4

[B] Prove that a continuous image of connected space is connected. 4

OR

Q: 4. Prove that the space $(R, \mathcal{U})$ is connected 8