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V.P.& R.P.T.P.Science College.Vallabh Vidyanagar.
Internal Test
B.Sc. Semester VI
US06CMTH04 (Abstract Algebra -2)
Thursday , 13^{th} March 2014
3.30 p.m. to 5.00 p.m.

Maximum Marks: 30

Que.1 Answer the following (Any three) 6

(1) Let f be a ring homomorphism ,then prove that f is one-one iff $Kerf = \{0\}$.

(2) Let R be a ring.Then prove that
(i) $a(-b) = -(ab)$, $\forall\, a, b \in R$. (ii) $a(b-c) = ab - ac$, $\forall\, a, b, c \in R$.

(3) Let $R = C[0,1]$,then prove that $I = \{\ x\ /x \in R,\ x(1/2) = 0\}$ is an ideal in R .

(4) Find Z_6/I , where $I = \{\bar{0}, \bar{2}, \bar{4}\}$.

(5) Show that 1+i is irreducible in Z+iZ .

(6) Find gcd of 2+3i and 4+7i in Z+iZ .

Que.2 (a) Let $C[0,1]$ be the set of real valued continuous function on [0,1].
Define + , *and* $\cdot$ in C[0,1] by
$(x+y)(t) = x(t) + y(t)$; $(xy)(t) = x(t)y(t)$, $\forall\, x, y \in R$, $t \in [0,1]$.
Prove that the set $C[0,1]$ forms a ring. Is it an integral domain ? Is it field ? 5

(b) Prove that every field is an integral domain . 3

OR

Que.2 (a) State and prove Cayley's theorem for rings. 5

(b) Prove that the only isomorphism of $\mathbb{Q}$ onto $\mathbb{Q}$ is the identity map $I_{\mathbb{Q}}$. 3

Que.3 (a) Prove that an ideal M in Z is a maximal ideal iff $M = pZ$, where p is a prime. 6

(b) Let $f : R \to R'$ be ring homomorphism ,then prove that Kerf is an ideal in R. 2

OR

Que.3 (a) Prove that every field is a simple ring . Does the converse hold ? Verify it. 8

Que.4 (a) Show that the ring of Gaussian integers is Euclidean domain . 5

(b) Let$R = \{a + b\sqrt{-5}/a, b \in Z\}$. Show that $1 + 2\sqrt{-5}$ and 3 are relatively prime. 3

OR

Que.4 (a) Prove that every prime element is irreducible in integral domain with unit element 1.
Does the converse hold ? Verify it. 6

(b) Let $R = Z$, $n \in Z$, $n > 1$.Then prove that n is irreducible iff n is prime number. 2

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