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V.P.& R.P.T.P.Science College.Vallabh Vidyanagar.
B.Sc. (Semester - V) Internal Test
US05CMTH04 (Abstract Algebra - 1)

Date. 4/10/2016 ; Tuesday 11.00 a.m. to 12.30 p.m. Maximum Marks: 25

Que.1 Fill in the blanks. 3

(1) Multiplicative inverse of 5 in ${Z_7}^*$ is ....

(a) 3 (b) 6 (c) 2 (d) 1

(2) .... is generator of group ${Z_5}^*$.

(a) $\bar{0}$ (b) $\bar{1}$ (c) $\bar{4}$ (d) $\bar{2}$

(3) Every cyclic group of .... order is simple group .
(a) 4 (b) prime (c) 6 (d) 1

Que.2 Answer the following (Any Two) 4

(1) Prove that every group has unique unit element .

(2) Find all generators of group $\{...\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, ...\}$.

(3) Prove that homomorphic image of abelian group is also abelian .

Que.3 (a) Prove that $(G, *)$ is a commutative group .Where G is a set of all subsets of $\mathbb{R}$ and $*$ defined as $A * B = (A - B) \cup (B - A)$; $\forall\ A, B \in G$. 4

(b) Let H and K be subgroups of group G , if HK is subgroup of G then prove that $HK = KH$. 2

OR

Que.3 (a) Let H and K be finite subgroups of group G such that HK is a subgroup of G.Then prove that $O(HK) = \dfrac{O(H)O(K)}{O(H \cap K)}$. 4

(b) Prove that fourth root of unity forms a group . 2

Que.4 (a) Prove that any subgroup of a cyclic group is also cyclic group . 4

(b) Prove that an infinite cyclic group has exactly two generators . 2

OR

Que.4 (a) Let G be a finite cyclic group of order n . Then prove that G has unique subgroup of order d for every divisor d of n . 4

(b) If G is a finite group and H a subgroup of G , then prove that $O(G) = O(H)(G : H)$. 2

Que.5 (a) State and prove First isomorphism theorem . 4

(b) Prove that a subgroup H is normal in group G iff $xH = Hx\ \forall\ x \in G$. 2

OR

Que.5 (a) Let $G' = \{1, \rho, \rho^2, ..., \rho^{n-1}\}$ be the multiplicative group of n^{th} root of unity, where $\rho = e^{2\pi i/n}$. Then prove that $Z_n \simeq G'$. 4

(b) Prove that any finite cyclic group of order n is isomorphic to Z_n . 2