EXTRA

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Internal Test: 2017-18

Subject : Mathematics US05CMTH01 Max. Marks : 25

Real Analysis-I

Date: 03/10/2017 Timing: 11.00 am - 12.30 pm

Q: 1. Answer the following by choosing correct answers from given choices. **3**

[1] The greatest member of a set S, if exists, is
[A] the supremum of S [B] the infimum of S [C] not unique [D] none

[2] The interior of the set of integers is
[A] N [B] Q [C] R [D] ϕ

[3] If $\lim_{x \to a} f(x)$ exists but $f(a)$ does not exist then f posseses a discontinuity of
[A] removable type [B] first type [C] second type [D] first type from left

Q: 2. Answer ANY TWO of the following. **4**

[1] Find the g.l.b and greatest member of $\left\{ \frac{5}{n^3} \, / n \in N \right\}$ if they exist.

[2] Give an example of a set which has no infimum but has supremum which is not a member of the set.

[3] Examine the following function for continuity at $x = 0$

$$f(x) = \begin{cases} \dfrac{xe^{\frac{1}{x}}}{1+e^{\frac{1}{x}}} & \text{when} \quad x \neq 0 \\ 0, & \text{when} \quad x = 0 \end{cases}$$

Q: 3. State the Least Upper Bound property of R and prove that the field of rational numbers is not order complete. **6**

OR

Q: 3 [A] State and prove the addition formulae for exponential function. **3**

[B] State and prove the Archimedean property of R and deduce that for any real number c there exists a positive integer n such that $n > c$. **3**

Q: 4 [A] Prove that derived set of a set is closed. **3**

[B] Prove that arbitrary union of open sets is open. **3**

OR

Q: 4 [A] If S and T are sets of real numbers then prove the following
(i) $S \subset T \Rightarrow S' \subset T'$ (ii) $(S \cup T)' = S' \cup T'$ **3**

[B] Define Interior point of a set and show that the interior of a set is an open set. **3**

Q: 5 [A] Prove that the function f defined on $\mathbb{R}$ as follows is discontinuous at every point.

$$f(x) = \begin{cases} 1 & \text{when x is irrational} \\ -1 & \text{when x is rational} \end{cases}$$

3

[B] If a function is continuous on a closed interval $[a, b]$, then it attains its bounds at least once in $[a, b]$. **3**

OR

Q: 5. If a function f is continuous on $[a, b]$ and $f(a)$ and $f(b)$ are of opposite signs, then prove that there exists at least one point $\alpha \in (a, b)$ such that $f(\alpha) = 0$. **6**