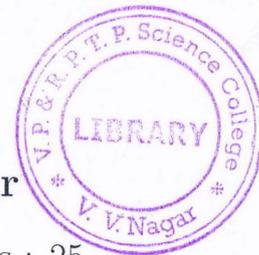


EXTRA



V.P. & R.P.T.P. Science College, V.V. Nagar

Internal Test: 2017-18

Subject : Mathematics

US05CMTH01

Max. Marks : 25

Real Analysis-I

Date: 03/10/2017

Timing: 11.00 am - 12.30 pm

Q: 1. Answer the following by choosing correct answers from given choices. 3

- [ 1 ] The greatest member of a set  $S$ , if exists, is  
[A] the supremum of  $S$  [B] the infimum of  $S$  [C] not unique [D] none
- [ 2 ] The interior of the set of integers is  
[A]  $\mathbb{N}$  [B]  $\mathbb{Q}$  [C]  $\mathbb{R}$  [D]  $\emptyset$
- [ 3 ] If  $\lim_{x \rightarrow a} f(x)$  exists but  $f(a)$  does not exist then  $f$  possesses a discontinuity of  
[A] removable type [B] first type [C] second type [D] first type from left

Q: 2. Answer ANY TWO of the following. 4

- [ 1 ] Find the g.l.b and greatest member of  $\left\{ \frac{5}{n^3} / n \in \mathbb{N} \right\}$  if they exist.
- [ 2 ] Give an example of a set which has no infimum but has supremum which is not a member of the set.
- [ 3 ] Examine the following function for continuity at  $x = 0$

$$f(x) = \begin{cases} \frac{xe^{\frac{1}{x}}}{1 + e^{\frac{1}{x}}} & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$$

Q: 3. State the Least Upper Bound property of  $\mathbb{R}$  and prove that the field of rational numbers is not order complete. 6

OR

Q: 3 [A] State and prove the addition formulae for exponential function. 3

[ B ] State and prove the Archimedean property of  $\mathbb{R}$  and deduce that for any real number  $c$  there exists a positive integer  $n$  such that  $n > c$ . 3

Q: 4 [A] Prove that derived set of a set is closed. 3

[ B ] Prove that arbitrary union of open sets is open. 3

OR

Q: 4 [A] If  $S$  and  $T$  are sets of real numbers then prove the following

(i)  $S \subset T \Rightarrow S' \subset T'$  (ii)  $(S \cup T)' = S' \cup T'$

3

[ B] Define Interior point of a set and show that the interior of a set is an open set.

3

Q: 5 [A] Prove that the function  $f$  defined on  $\mathbb{R}$  as follows is discontinuous at every point.

$$f(x) = \begin{cases} 1 & \text{when } x \text{ is irrational} \\ -1 & \text{when } x \text{ is rational} \end{cases}$$

3

[ B] If a function is continuous on a closed interval  $[a, b]$ , then it attains its bounds at least once in  $[a, b]$ .

3

OR

Q: 5. If a function  $f$  is continuous on  $[a, b]$  and  $f(a)$  and  $f(b)$  are of opposite signs, then prove that there exists at least one point  $\alpha \in (a, b)$  such that  $f(\alpha) = 0$ .

6

