V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2017-18

Subject : Mathematics US05CMTH02 Max. Marks : 25

Real Analysis-II

Date: 04/10/2017 Timing: 11.00 am - 12.30 pm

Q: 1. Answer the following by choosing correct answers from given choices. **3**

[1] A sequence $\{S_n\}$; where

$$S_n = \begin{cases} 2 & ; \text{ if } n = 1 \text{ or even} \\ p & ; \text{ where } p \text{ is the smallest prime factor of n.} \end{cases}$$

is

[A] convergent [B] divergent [C] oscillates finitely [D] oscillates infinitely

[2] A positive term series $\sum_{n=1}^{\infty} u_n$ is convergent if

[A] $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = 0$ [B] $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} > 1$ [C] $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} < 1$ [D] none

[3] $\lim_{(x,y)\to(0,0)} \dfrac{x \sin(x^2+y^2)}{x^2+y^2} =$

[A] 0 [B] 1 [C] 2 [D] 3

Q: 2. Answer ANY any TWO of the following. **4**

[1] Prove that every convergent sequence is bounded.

[2] Show that the necessary condition for convergence of an infinite series $\sum_{n=1}^{\infty} u_n$ is that $\lim_{n\to\infty} u_n = 0$

[3] Show that the following function is discontinuous at $(2,3)$

$$f(x,y) = \begin{cases} 2x + 3y^3 ; & \text{when } (x,y) \neq (2,3) \\ 0 ; & \text{when } (x,y) = (2,3) \end{cases}$$

Q: 3 [A] State and prove the Bolzano-Weicerstarss theorem for sequence **3**

[B] Show that $\lim_{n\longrightarrow\infty} \sqrt[n]{n} = 1$ **3**

OR

Q: 3 [A] If $\{a_n\}$ and $\{b_n\}$ are two sequences such that $\lim_{n\longrightarrow\infty} a_n = a$ and $\lim_{n\longrightarrow\infty} b_n = b$, then prove that $\lim_{n\longrightarrow\infty} (a_n + b_n) = a + b$ **3**

[B] Prove that a sequence $\{S_n\}$ defined by the recursion formula $S_{n+1} = \sqrt{7+S_n}$, where $S_1 = \sqrt{7}$, converges to the positive root of $x^2 - x - 7 = 0$ 3

Q: 4 [A] State and prove *Cauchy's* general principle for convergence of a series. 3

[B] Show that the positive term series $1 + r + r^2 + r^3 + \ldots$ is convergent for $r < 1$ and diverges to $+\infty$ for $r \geqslant 1$. 3

OR

Q: 4. State and prove the comparision test of second type. 6

Q: 5 [A] Show that the following function is discontinuous at $(0, 0)$ 3

$$f(x,y) = \begin{cases} \frac{x^2y}{x^3+y^3} & , \text{ if } (x,y) \neq (0,0) \\ 0 & , \text{ if } (x,y) = (0,0) \end{cases}$$

[B] Show that $\lim\limits_{(x,y)\to(0,0)} xy\dfrac{x^2-y^2}{x^2+y^2} = 0$ 3

OR

Q: 5. Show that $\frac{\partial^2 \theta}{\partial x \partial y} = -\frac{\cos 2\theta}{r^2}$ 6