EXTRA

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Internal Test: 2017-18

Subject : Mathematics | US05CMTH03 | Max. Marks : 25

Metric Spaces

Date: 05/10/2017 | Timing: 11.00 am - 12.30 pm

Q: 1. Answer the following by choosing correct answers from given choices. **3**

[1] The set of all cluster points of $(1, 2)$ is
[A] $[1, 2]$ [B] $[1, 2)$ [C] $(1, 2]$ [D] $(1, 2)$

[2] In the metric space $M = [0, 1]$ with usual metric , $B[\frac{1}{4}, 1] =$
[A] $[0, 1]$ [B] $[\frac{1}{4}, 1]$ [C] $[0, \frac{1}{4}]$ [D] $(0, 1)$

[3] subset $(0, \infty)$ of R^1 is
[A] bounded
[B] totally bounded
[C] neither bounded nor totally bounded
[D] none

Q: 2. Answer ANY TWO of the following. **4**

[1] Show that $\rho : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$, defined by $\rho(x, y) = |x - y|$, is a metric on $\mathbb{R}$

[2] Prove that in any metric space (M, ρ), both M and ϕ are open sets.

[3] Prove that every contraction mapping is continuous.

Q: 3. Define limit of a function. Also prove that **6**
$\lim_{x \longrightarrow a} [f(x).g(x)] = \lim_{x \to a} f(x). \lim_{x \to a} g(x)$

OR

Q: 3 [A] Prove that if $\{s_n\}_{n=1}^{\infty}$ is a convergent sequence of points in a metric space (M, ρ) then $\{s_n\}_{n=1}^{\infty}$ is Cauchy. Is the converse true? Justify. **3**

[B] For $P(x_1, y_1)$ and $Q(x_2, y_2)$ in $\mathbb{R}^2$ define $\tau : \mathbb{R}^2 \times \mathbb{R}^2 \longrightarrow \mathbb{R}$ by

$$\tau(P, Q) = max(|x_1 - x_2|, |y_1 - y_2|)$$

Show that τ is a metric on $\mathbb{R}^2$ **3**

Q: 4 [A] If F_1 and F_2 are closed subsets of the metric space M, then prove that $F_1 \cup F_2$ is also closed. **3**

[B] If A_1 and A_2 are connected subsets of a metric space M and if $A_1 \cap A_2 \neq \phi$, then prove that $A_1 \cup A_2$ is also connected. **3**

OR

Q: 4 [A] Prove that $(0, \infty)$ and $(0, 1)$ are homeomorphic. **3**

[B] Prove that Every open subset G of $\mathbb{R}$ can be written $G = \bigcup I_n$, where $I_1, I_2, I_3, \ldots$ are a finite number or a countable number of open intervals which are mutually disjoint. **3**

Q: 5 [A] Prove that every finite subset of a metric space M is totally bounded. **3**

[B] If (M, ρ) is a complete metric space and A is a closed subset of M, then prove that (A, ρ) is also complete. **3**

OR

Q: 5. State and prove Picard's fixed point theorem. **6**