

EXTRA

V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2017-18

Subject : Mathematics

US05CMTH03

Max. Marks : 25

Metric Spaces

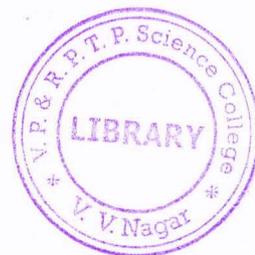
Date: 05/10/2017

Timing: 11.00 am - 12.30 pm

Q: 1. Answer the following by choosing correct answers from given choices.

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- [1] The set of all cluster points of $(1, 2)$ is
 [A] $[1, 2]$ [B] $[1, 2)$ [C] $(1, 2]$ [D] $(1, 2)$
- [2] In the metric space $M = [0, 1]$ with usual metric, $B[\frac{1}{4}, 1] =$
 [A] $[0, 1]$ [B] $[\frac{1}{4}, 1]$ [C] $[0, \frac{1}{4}]$ [D] $(0, 1)$
- [3] subset $(0, \infty)$ of R^1 is
 [A] bounded
 [B] totally bounded
 [C] neither bounded nor totally bounded
 [D] none



Q: 2. Answer ANY TWO of the following.

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- [1] Show that $\rho : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$, defined by $\rho(x, y) = |x - y|$, is a metric on \mathbb{R}
- [2] Prove that in any metric space (M, ρ) , both M and ϕ are open sets.
- [3] Prove that every contraction mapping is continuous.

Q: 3. Define limit of a function. Also prove that

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$$\lim_{x \rightarrow a} [f(x).g(x)] = \lim_{x \rightarrow a} f(x). \lim_{x \rightarrow a} g(x)$$

OR

Q: 3 [A] Prove that if $\{s_n\}_{n=1}^\infty$ is a convergent sequence of points in a metric space (M, ρ) then $\{s_n\}_{n=1}^\infty$ is Cauchy. Is the converse true? Justify.

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[B] For $P(x_1, y_1)$ and $Q(x_2, y_2)$ in \mathbb{R}^2 define $\tau : \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow \mathbb{R}$ by

$$\tau(P, Q) = \max(|x_1 - x_2|, |y_1 - y_2|)$$

Show that τ is a metric on \mathbb{R}^2

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Q: 4 [A] If F_1 and F_2 are closed subsets of the metric space M , then prove that $F_1 \cup F_2$ is also closed.

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[B] If A_1 and A_2 are connected subsets of a metric space M and if $A_1 \cap A_2 \neq \phi$, then prove that $A_1 \cup A_2$ is also connected.

3

OR

Q: 4 [A] Prove that $(0, \infty)$ and $(0, 1)$ are homeomorphic. 3

[B] Prove that Every open subset G of \mathbb{R} can be written $G = \bigcup I_n$, where I_1, I_2, I_3, \dots are a finite number or a countable number of open intervals which are mutually disjoint. 3

Q: 5 [A] Prove that every finite subset of a metric space M is totally bounded. 3

[B] If (M, ρ) is a complete metric space and A is a closed subset of M , then prove that (A, ρ) is also complete. 3

OR

Q: 5. State and prove Picard's fixed point theorem. 6

