V.P.& R.P.T.P.Science College.Vallabh Vidyanagar.
B.Sc. (Semester - V) Internal Test
US05CMTH04 (Abstract Algebra - 1)
Date. 6/10/2017 ; Friday 11.00 a.m. to 12.30 p.m. Maximum Marks: 25

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Que.1 Fill in the blanks. 3

(1) .... is subgroup of group $\{z \in \mathbb{C} \ / \ |z| = 1\}$.

(a) $\{\pm 1, \pm i\}$ (b) $\{\pm 2, \pm i\}$ (c) $\{1, i\}$ (d) $\{\pm i\}$

(2) Cyclic group with one generator has at most .... elements.

(a) 0 (b) 3 (c) 1 (d) 2

(3) Define $f : R^* \to R^*$ by $f(x) = 1/x$ then Ker $f =$ ....

(a) 1 (b) 0 (c) $\{1\}$ (d) $\{\pm 1\}$

Que.2 Answer the following (Any Two) 4

(1) Prove right cancelation law for group.

(2) Give an example of finite cyclic group.Verify it.

(3) Prove that any group of order 4 is abelian.

Que.3 (a) Prove that $(Q - \{1\}, *)$ is a group , where $*$ is defined as $a * b = a + b - ab$,$\forall\, a, b \in Q - \{1\}$. 3

(b) Prove that the product of two subgroup of group is a subgroup iff they commute with each other. 3

OR

Que.3 (a) Let H be a finite subset of group G such that $ab \in H$ whenever $a, b \in H$.Then prove that H is a subgroup of G. 4

(b) For group G prove that $(ba)^{-1} = a^{-1}b^{-1}$ 2

Que.4 (a) Prove that any subgroup of a cyclic group is also cyclic group . 4

(b) Let H be a subgroup of G.Then prove that the number of left cosets of H in G is same as the number of right cosets of H in G. 2

OR

Que.4 (a) Prove that every subgroup of an infinite cyclic group is also an infinite cyclic group. 3

(b) Let G be a finite cyclic group of order n , then prove that G has $\phi(n)$ generators. 3

Que.5 (a) State and prove First and Third isomorphism theorem . 6

OR

Que.5 (a) Let $G = (a)$ be a finite cyclic group of order n .Then prove that the mapping $\theta : G \to G$ defined by $\theta(a) = a^m$ is an automorphism of G iff m is relatively prime to n . 4

(b) Prove that $\theta : Z \to Z$ defined by $\theta(n) = -n$ is an automorphism of Z. 2

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