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V.P.& R.P.T.P.Science College,Vallabh Vidyanagar.
B.Sc.(Semester - V) Internal Test
US05CMTH05 (Number Theory)

Date. 7/10/2017 ; Saturday 11.00 a.m. to 12.30 p.m. Maximum Marks: 25

Que.1 Fill in the blanks. 3

(1) If a/b then $(a\ ,\ b) =$ .... $\forall\ a\ ,\ b \in \mathbb{Z}$.

(a) a (b) $|a|$ (c) $|b|$ (d) b

(2) S(60) = ...

(a) 61 (b) 60 (c) 12 (d) 168

(3) Prove that every number containing more than three digits can be divided by 8 iff the number formed by .... digits can be divided by 8.

(a) last two (b) last three (c) first two (d) first three

Que.2 Answer the following (Any Two) 4

(1) Prove that $(a-s)/(ab+st) \Rightarrow (a-s)/(at+sb)$.
(2) Prove that $[x]+[y] \leq [x+y] \leq [x]+[y]+1$.
(3) Prove that the indeterminate equation $ax+by=c$ has solution iff d/c , where $(a,b)=d$.

Que.3 (a) Let g be a positive integer greater than 1 then prove that every positive integer a can can be written uniquely in the form $a = c_n g^n + c_{n-1} g^{n-1} + ... + c_1 g + c_0$, where $n \geq 0\ ,\ c_i \in \mathbb{Z}\ ,\ 0 \leq c_i < g\ ,\ c_n \neq 0$. 4

(b) If a is a composite number and q is its least positive divisor then prove that $q < \sqrt{a}$. 2

OR

Que.3 (a) If P_n is n^{th} prime number then prove that $P_n < 2^{2^n}$, $\forall\ n \in \mathbb{N}$. 4

(b) Prove that $(a,b) = (ka+b, b)$, for $k \in Z$. 2

Que.4 (a) Prove that any prime factor of M_p is greater than p . 4

(b) In usual notation prove that $\sum_{d/a} \mu(d) = 0$, if $a > 1$. 2

OR

Que.4 (a) Prove that $S(a) < a\sqrt{a}$, $\forall\ a > 2$. 3

(b) Prove that odd prime factor of $a^{2^n} + 1 (a > 1)$ is of the form $2^{n+1}t + 1$, for some integer t . 3

Que.5 (a) Prove that a general integer solution of $x^2+y^2+z^2=w^2$, $(x,y,z,w)=1$ is given by $x=(a^2-b^2+c^2-d^2)$, $y=2ab-2cd$, $z=2ad+2bc$, $w=a^2+b^2+c^2+d^2$. 5

(b) Solve the equation $525x+231y=24$ if possible . 1

OR

Que.5 (a) Prove that the positive integer solution of $x^{-1}+y^{-1}=z^{-1}$, $(x,y,z)=1$ has and must have the form $x=a(a+b)$, $y=b(a+b)$, $z=ab$, where $a\ ,\ b\ >\ 0$, $(a,b)=1$. 3

(b) Find general solution of equation $50x+45y+36z=10$. 3

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