V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2017-18

Subject : Mathematics US06CMTH01 Max. Marks : 25

Real Analysis - III

Date: 12/03/2018 Timing: 11:00 am - 12:30 pm

Q: 1. Answer the following by choosing correct answers from given choices. **3**

[1] In usual notations, the Lagrange's form of remainder in Maclaurin's theorem is

[A] $\frac{x^{n-1}(1-\theta)^{n-p}}{p(n-1)!}f^{(n)}(a+\theta x)$ [B] $\frac{x^{n}(1-\theta)^{n-x}}{p(n-1)!}f^{(n)}(a+\theta x)$

[C] $\frac{x^n}{n!}f^{(n)}(\theta x)$ [D] $\frac{x^n}{n!}f^{(n-1)}(\theta x)$

[2] If f has an extreme value at c then there is some $\delta > 0$, such that $\forall x \in (c-\delta, c+\delta)-\{c\}$

[A] $f(x)-f(c)$ keeps same sign [B] $f'(x)-f'(c)$ keeps same sign

[C] $f''(x)-f''(c)$ keeps same sign [D] none

[3] If f is a bounded function defined on $[a,b]$ then for a given $\epsilon > 0$ there is always a partition P of $[a,b]$ such that

[A] $\underline{\int_a^b} f.dx < L(P,f)+\epsilon$ [B] $\underline{\int_a^b} f.dx < L(P,f)-\epsilon$

[C] $\overline{\int_a^b} f.dx > U(P,f)+\epsilon$ [D] $\overline{\int_a^b} f.dx < U(P,f)-\epsilon$

Q: 2. Answer any TWO of the following. **4**

[1] Explain the geometric meaning of Rolle's theorem

[2] Show that, $f(x) = x^2 - 4x - 5$ has a minimum at 2

[3] Write any two refinements of a partition $\{1, 1.2, 1.3, 1.4, 1.5, 2\}$ of $[1,2]$

Q: 3 [A] State and prove Lagrange's Mean Value theorem **3**

[B] A twice differentiable function f is such that $f(a) = f(b) = 0$ and $f(c) > 0$ for $a < c < b$. Prove that there is at least one value ξ between a and b for which $f''(\xi) < 0$. **3**

OR

Q: 3 [A] Prove Taylor's theorem with Cauchy's form of remainder by taking the function

$$\phi(x) = f(x) + \frac{(a+h-x)}{1!}f'(x) + \frac{(a+h-x)^2}{2!}f''(x) + \cdots\cdots + \frac{(a+h-x)^{n-1}}{(n-1)!}f^{(n-1)}(x) + A(a+h-x)$$

3

[B] Show that $\dfrac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \cot\theta$, for some θ where $0 < \alpha < \theta < \beta < \dfrac{\pi}{2}$ 3

Q: 4 [A] If c is an interior point of the domain of a function f and $f'(c) = 0$ then prove that the function has maxima or minima at c according as $f''(c)$ is negative or positive 3

[B] Examine the function $(x-3)^5(x+1)^4$ for extreme values 3

OR

Q: 4 [A] Prove that if $f(c)$ is an extreme value of a function then $f'(c)$, if exists, is zero. 3

[B] Show that the maximum value of $\dfrac{\log x}{x}$ in $0 < x < \infty$ is $\dfrac{1}{e}$ 3

Q: 5. Prove that if f and g are bounded and integrable functions on $[a, b]$ and there exists a number $\lambda > 0$ such that $|g(x)| \geqslant \lambda, \forall x \in [a, b]$ then $\dfrac{f}{g}$ is also bounded and integrable on $[a, b]$ 6

OR

Q: 5 [A] Prove that a function f is integrable over $[a, b]$ iff there is a number I such that for any $\epsilon > 0$, $\exists$ a partition P of $[a, b]$ such that,

$$|U(P, f) - I| < \epsilon \text{ and } |I - L(P, f)| < \epsilon$$

3

[B] Show that x^2 is integrable on any interval $[0, k]$ 3