V.P.& R.P.T.P.Science College.Vallabh Vidyanagar.
Internal Test B.Sc. Semester VI
US06CMTH02 (Complex Analysis)
Tuesday , 13^{th} March 2018 11.00 a.m. to 12.30 p.m. Maximum Marks :25

Que.1 Fill in the blanks. 3

(1) $f(z) = (x^2 - y^2 - 2y) + i(2x - 2xy)$ can be expressed as $f(z) =$........................

(a) $\bar{z}^2 + 2z$ (b) $\bar{z}^2 + iz$ (c) $\bar{z}^2 - 2iz$ (d) $\bar{z}^2 + 2iz$

(2) Singular point of $f(z) = \dfrac{z^3 + i}{(z^2 + 3z + 2)}$ are $z =$ ....

(a) 1 , 2 (b) 1 , i (c) 1 , 3 , i (d) -1 , -2

(3) $exp(2 \pm 3\pi i) =$ ....

(a) $-e^2$ (b) e^2 (c) e^{-2} (d) $-e$

Que.2 Answer the following (Any Two) 4

(1) By using definition of limit prove that $\lim_{z \to z_0} (z^2 + c) = {z_0}^2 + c$, where c is complex constant.

(2) Prove that in domain D , v is harmonic conjugate of u iff $-u$ is harmonic conjugate of v .

(3) Solve $e^z = -\sqrt{3} + i$.

Que.3 (a) State and prove chain rule for differentiating composite functions. 4

(b) By using definition of limit prove that $\lim_{z \to (1-i)} (x + i(2x + y)) = 1 + i.$ 2

OR

Que.3 (a) Give an example of function such that its real and imaginary component have continuous partial derivative of all order at a point but the function is not differentiable at that point.Verify it. 4

(b) If $f(z) = \dfrac{x^3 y(y - ix)}{z(x^6 + y^2)}$, $z \neq 0$, $f(0) = 0$. Is $\lim_{z \to 0} f(z)$ exists ? 2

Que.4 (a) Let $f(z) = u(x, y) + iv(x, y)$ and $f'(z)$ exist at $z_0 = x_0 + iy_0$.Prove that the first order partial derivatives of u and v must exist at (x_0, y_0) and they satisfies the Cauchy-Reimann equations $u_x = v_y$; $u_y = -v_x$ *at* (x_0, y_0). Does the converse of above result holds? Verify it. 6

OR

Que.4 (a) Find harmonic conjugate of $u(x, y) = \dfrac{x}{x^2 + y^2}$. 3

(b) Prove that $f'(z)$ and $f''(z)$ exist everywhere and find $f''(z)$ for $f(z) = cosxcoshy - isinxsinhy$. 3

Que.5 (a) Prove that $cosz_1 - cosz_2 = -2sin\left(\dfrac{z_1 + z_2}{2}\right) sin\left(\dfrac{z_1 - z_2}{2}\right)$. 3

(b) Find all roots of $coshz = 1/2$. 3

OR

Que.5 (a) Prove that $|sinhx| \leq |coshz| \leq coshx.$ 3

(b) Prove that $\cos^{-1} z$ is multiple valued function . 3

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