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Internal Test: 2017-18

Subject : Mathematics   US06CMTH03   Max. Marks : 25

Topology

Date: 1~~3~~4/03/2018   Timing: 11:00 am - 12:30 pm

Q: 1. Answer the following by choosing correct answers from given choices. **3**

[1] In a topological space $(X, \mathcal{T})$, a neighbourhood of a point is
[A] $\mathcal{T}$-open [B] $\mathcal{T}$-closed [C] either open or closed [D] none

[2] If A is a dense subset of a topological space $(X, \mathcal{T})$ then
[A] $A' = X$ [B] $A = X$ [C] $A^- = X$ [D] none

[3] If I is an open interval then the subspace $(I, \mathcal{U}_I)$ and $(R, \mathcal{U})$
[A] both are compact [B] are homeomorphic
[C] both are bounded [D] none

Q: 2. Answer any TWO of the following. **4**

[1] Give an example of a Door Space

[2] Find the $\mathcal{U}$-closure of each of the following subsets of $\mathbb{R}$
(a) $\mathbb{R}$ (b) $\emptyset$

[3] Let $f : [1, 10] \to R$ be continuous on $[1, 10]$. Is $f([1, 10])$ connected?

Q: 3 [A] Let J be the set of all integers and $\mathcal{J}$ be a collection of subsets G of J where $G \in \mathcal{J}$ whenever $G = \emptyset$ or $G \neq \emptyset$ and $p, p \pm 2, p \pm 4, ..., p \pm 2n, ...$ belong to G whenever $p \in G$.Prove that $\mathcal{J}$ is a topology for J **3**

[B] Let $(X, \mathcal{T})$ be a topological space and let $J_n = \{1, 2, 3, ..., n\}$. If $F_1, F_2, ..., F_n$ are $\mathcal{T}$-closed subsets of X then prove that $\bigcup\{F_i / \ i \in J\}$ is a $\mathcal{T}$-closed set **3**

OR

Q: 3 [A] If $\{G_\alpha \ / \ \alpha \in \Lambda\}$ is a collection of $\mathcal{U}$-open subsets of $\mathbb{R}$ then prove that $\bigcup\{G_\alpha \ / \ \alpha \in \Lambda\}$ is a $\mathcal{U}$-open set **3**

[B] Let $(X, \mathcal{T})$ be a topological space and let A be a subset of X. Prove that A is $\mathcal{T}$-open set iff A contains a $\mathcal{T}$-neighbourhood of each of its points **3**

Q: 4 [A] Let $(X, \mathcal{T})$ be a topological space and $A \subset X$. Prove the following
(i) A is $\mathcal{T}$-open iff $Int(A) = A$ (ii) $Int(A)$ is the largest open subset of A **3**

[B] Let $(X, \mathcal{T})$ be a topological space and let A be a subset of X and A' be the set of all cluster points of A. Prove that A is $\mathcal{T}$-closed iff $A' \subset A$ **3**

OR

Q: 4 [A] Let $(X, \mathcal{T})$ be a topological space and A be a subset of X. Prove that $A \cup A'$ is $\mathcal{T}$-closed **3**

[B] Let $(X, \mathcal{T})$ be a topological space and let A be a subset of X. Then prove that $A^{-} = A \cup A'$. **3**

Q: 5. Prove that the space $(R, \mathcal{U})$ is connected. **6**

OR

Q: 5 [A] Prove that if $(X, \mathcal{T})$ is disconnected then there is a nonempty proper subset of X that is both $\mathcal{T}$-open and $\mathcal{T}$-closed. **3**

[B] Assuming that connectedness is a topological property prove that $(R, \mathcal{U})$ and $(R, \mathcal{G})$ are not homeomorphic where $\mathcal{U}$ is usual topology for R and $\mathcal{G}$ is defined as follows
$G \in \mathcal{G}$ if either G empty or it is a nonempty subset of R such that for every $p \in G$ there is some $H = \{x \in R / a \leqslant x < b\}$ for $a < b$ such that $p \in H \subset G$. **3**