V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2018-19

Subject : Mathematics | US05CMTH02 | Max. Marks : 50

Real Analysis-II

Date: 01/10/2018 | Timing: 10.00 am - 12.00 pm

Q: 1. Answer the following by choosing correct answers from given choices. **8**

[1] The sequence $\{S_n\}_{n=1}^{\infty}$, where $S_n = (-1)^n \left(1 + \frac{1}{n}\right)$
[A] is convergent [B] oscillates finitely [C] oscillates infinitely [D] is divergent

[2] Every convergent sequence is
[A] oscillating [B] bounded [C] unbounded [D] none

[3] A positive term series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if and only if
[A] $p < 1$ [B] $p > 1$ [C] $p \leqslant 1$ [D] $p \geqslant 1$

[4] The series $\sum_{n=1}^{\infty} \frac{n+1}{n}$
[A] converges to 1 [B] converges to 2 [C] converges to 3 [D] none

[5] $\lim_{(x,y)\to(0,0)} \frac{x \sin(x^2+y^2)}{x^2+y^2} =$
[A] 0 [B] 1 [C] 2 [D] 3

[6] $\lim_{x\to 1} \lim_{y\to -1} \frac{4x^3y^2}{x^2+y^2} =$
[A] 1 [B] 2 [C] 3 [D] none

[7] The necessary condition for a function f to have an extreme value at $(2, 4)$ is
[A] $f_x(2,4) = 0,\ f_y(2,4) \neq 0$ [B] $f_x(2,4) \neq 0,\ f_y(2,4) = 0$
[C] $f_x(2,4) \neq 0,\ f_y(2,4) \neq 0$ [D] $f_x(2,4) = 0,\ f_y(2,4) = 0$

[8] For a function f, if $f_x(a,b) < f_y(a,b)$ then at (a,b), f has
[A] no extreme value [B] a minimum [C] a maximum [D] an extreme value

Q: 2. Answer any FIVE of the following. **10**

[1] Define : (i) Bounded Sequence (ii) Convergence of a sequence

[2] Show that $\lim_{n\longrightarrow\infty} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2$

[3] If $\sum_{n=1}^{\infty} u_n = u$ and $\sum_{n=1}^{\infty} v_n = v$ then prove that $\sum_{n=1}^{\infty} (u_n - v_n) = u - v$

[4] Test $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{n^{-1}}$ for convergence.

[5] Evaluate : $\lim\limits_{(x,y)\to(3,1)} \frac{\tan^{-1}(xy-3)}{\tan^{-1}(2xy-6)}$

[6] Evaluate : $\lim\limits_{(x,y)\to(1,1)} \dfrac{e^{(x-y)}-1}{x-y}$

[7] Can a function $f(x,y) = x^2+5xy+y^2$ have an extreme value at $(1,1)$? Why?

[8] State Maclaurin's theorem

Q: 3 [A] State and prove the Bolzano-Weierstarss theorem for sequence 5

[B] Show that the sequence $\{S_n\}$ where $S_n = 1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}$ cannot converge. 3

OR

Q: 3 [A] If a sequence of closed intervals $[a_n, b_n]$ is such that each member $[a_{n+1}, b_{n+1}]$ is contained in the preceeding one $[a_n, b_n]$ and $\lim\limits_{n\to\infty} (b_n - a_n) = 0$ then prove that there is one and only one point common to all the intervals of the sequence. 5

[B] If $\lim\limits_{n\to\infty} a_n = a$ for a sequence $\{a_n\}$ such that $a_n \geqslant 0$ then show that $a \geqslant 0$ 3

Q: 4 [A] State and prove the comparision test of first type. 5

[B] Show that a positive term series converges *iff* the sequence of its partial sums is bounded above. 3

OR

Q: 4 [A] State and prove *Cauchy's* general principle for convergence of a series. 5

[B] Show that the series $\frac{1}{(\log 2)^p} + \frac{1}{(\log 3)^p} + \cdots + \frac{1}{(\log n)^p} + \ldots$ diverges for $p > 0$ 3

Q: 5 [A] By using the definition of limit prove that : $\lim\limits_{(x,y)\to(1,2)} (x^2+2y) = 5$ 5

[B] Show that $\lim\limits_{(x,y)\to(0,0)} xy\dfrac{x^2-y^2}{x^2+y^2} = 0$ 3

OR

Q: 5. If $z = f(x,y)$ is a function of independent variables x, y and if x, y are changed to new independent variables u, v by the substitution $x = \phi(u,v)$; $y = \psi(u,v)$, then express the derivatives of z with respect to x, y in terms of u, v and the derivatives of z with respect to u, v. 8

Q: 6 [A] State and prove Taylor's theorem 5

[B] show that $2x^4 - 3x^2y + y^2$ has neither a maximum nor a minimum at $(0,0)$. 3

OR

Q: 6 [A] Find the expansion of $\sin x \sin y$ about $(0,0)$ upto and including and the terms of fourth degree in (x,y). Also compare the result with that you get by multiplying the series for $\sin x$ and $\sin y$. 5

[B] A rectangular box open at the top is to have a volume of $32m^3$. Find the dimensions of box so that the total surface area is minimum. 3