V.P.& R.P.T.P.Science College.Vallabh Vidyanagar.
B.Sc. (Semester - V) Internal Test
US05CMTH04 (Abstract Algebra - 1)
Date. 5/10/2018 ; Friday 10.00 a.m. to 12.00 noon Maximum Marks: 50

Que.1 Fill in the blanks. 8

(1) In group $(Z_7^*, \cdot)$, $\bar{6}^{-1} =$ ....
(a) $\bar{6}$ (b) $\bar{3}$ (c) $\bar{4}$ (d) $\bar{2}$

(2) Let G be a group ,an element $a \in G$ is called idempotent if $a^2 =$ ....
(a) 0 (b) a (c) e (d) 1

(3) .... is generator of group Z_5^*.
(a) $\bar{3}$ (b) $\bar{1}$ (c) $\bar{4}$ (d) $\bar{5}$

(4) Cyclic group of order 13 has only .... generator .
(a) 13 (b) 11 (c) 12 (d) 2

(5) Define $f : R^* \to R^*$ by $f(x) = x^2$ then Ker $f =$ ....
(a) 0 (b) ± 1 (c) 1 (d) $\{\pm 1\}$

(6) .... is quotient group of Z_{12} .
(a) Z_2 (b) Z_8 (c) Z_{10} (d) Z_9

(7) If $f : A \to B$ is one-one homomorphism but not onto then $A \simeq$ ....
(a) $f(AB)$ (b) $f(B)$ (c) $f(A)$ (d) B

(8) Ker $\varepsilon =$ ....
(a) A_n (b) e (c) ± 1 (d) S_n

Que.2 Answer the following (Any Five) 10

(1) Prove that fourth root of unity forms a group.
(2) Prove that the product of two subgroup of group is a subgroup if they commute with each other.
(3) Find all right cosets of $-3\mathbb{Z}$ in $\mathbb{Z}$.
(4) Prove that an infinite cyclic group has exactly two generators.
(5) Prove that the mapping $\theta : \mathbb{R} \to \mathbb{R}^+$ defined by $\theta(a) = 2^a$ is onto isomorphism .
(6) Let G be a group and $x \in G$ be a fixed element . Then prove that the mapping $i_x : G \to G$ defined by $i_x(a) = xax^{-1}$ is an automorphism of G.
(7) Prove that $O(A_n) = \dfrac{n!}{2}$.
(8) If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 1 & 5 \end{pmatrix}$; $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 4 & 5 & 2 \end{pmatrix}$ then find $\varepsilon(\tau\sigma)$, $\varepsilon(\sigma\tau)$.

Que.3 (a) Is $(G, \cdot)$ forms a Group ? Is it Commutative ? Verify it , where G is set of all 2×2 non singular real matrices . 5

(b) Let H be a finite subset of group G such that $ab \in H$ whenever $a, b \in H$.Then prove that H is a subgroup of G. 3

OR

Que.3 (c) Let H and K be finite subgroups of group G such that HK is a subgroup of G.Then prove that $O(HK) = \dfrac{O(H)O(K)}{O(H \cap K)}$. 5

(d) Prove that $(G, *)$ is a group ,where G is a set of all subsets of $\mathbb{R}$ and operation $*$ defined as $A * B = (A - B) \cup (B - A) \quad \forall A, B \in G$. 3

Que.4 (a) Prove that any subgroup of a cyclic group is also cyclic group . Also prove that every subgroup of an infinite cyclic group is infinite cyclic group. 5

(b) Let G be a finite cyclic group of order n.Then prove that G has unique subgroup of order d for every divisor d of n . 3

OR

Que.4 (c) Let G be a finite cyclic group of order n , then prove that G has $\phi(n)$ generators . 5

(d) Let H be a subgroup of group G.Then prove that G is the union of all right cosets of H in G . Also prove that two distinct right cosets of H in G are disjoint. 3

Que.5 (a) State and prove First and Third isomorphism theorem . 8

OR

Que.5 (b) Let $G = (a)$ be a finite cyclic group of order n .Then prove that the mapping $\theta : G \to G$ defined by $\theta(a) = a^m$ is an automorphism of G iff m is relatively prime to n . 5

(c) Prove that $\mathbb{Z}_n \simeq n^{th}$ root of unity .

Que.6 (a) If G is a direct product of subgroups H and K ,then prove that G is isomorphic to the external direct product of H and K . 4

(b) Let $G = H \times K$ be external direct product of H and K ,then prove that $G/K' \simeq H$, where $K' = \{(e_H, k)/k \in K\}$. 4

OR

Que.6 (c) Prove that G is direct product of subgroups H and K iff (i) every $x \in G$ can be uniquely expressed as $x = hk$, $h \in H$, $k \in K$ (ii) $hk = kh$, $h \in H$, $k \in K$. 5

(d) Prove that every permutation can be expressed as a product of disjoint cycles . 3