V.P.& R.P.T.P.Science College, Vallabh Vidyanagar.
B.Sc.(Semester - V) Internal Test
US05CMTH05 (Number Theory)
Date. 6/10/2018 ; Saturday 10.00 a.m. to 12.00 noon Maximum Marks: 50

Que.1 Fill in the blanks. 8

(1) $(a\ ,\ 0) =$ .... , $\forall\ a \in \mathbb{Z}$
(a) $-a$ (b) $|a|$ (c) a (d) 0

(2) $(525, 231) =$ ....
(a) 10 (b) 31 (c) 21 (d) 7

(3) If a is square number then S(a) is ....
(a) even (b) odd (c) prime (d) 0

(4) .... is Fermat's number .
(a) 100 (b) 116 (c) 327 (d) 257

(5) 765432 is not divisible by ....
(a) 7 (b) 3 (c) 4 (d) 9

(6) $\phi(m) + S(m) = mT(m)$ iff m is ....
(a) not prime (b) odd (c) even (d) prime

(7) $\phi(m) \leq$ .... , $\forall\ m > 1$.
(a) m-1 (b) m (c) m+1 (d) m - 2

(8) $2x + 7y \equiv 5(mod\ 12)$ has only .... solutions.
(a) 1 (b) 2 (c) 12 (d) 5

Que.2 Answer the following (Any Five) 10

(1) Discuss Euclidean algorithm for finding gcd of two numbers .
(2) Prove that $(a + b)[a, b] = b[a, a + b]$, $\forall\ a\ ,\ b > 0$.
(3) Prove that two distinct Fermat's numbers are relatively prime .
(4) Prove that $u_{n+1}^2 = u_n^2 + 3u_{n-1}^2 + 2[u_{n-2}^2 + u_{n-3}^2 + \cdots + u_1]$.
(5) Find positive integer solution of $7x + 19y = 213$.
(6) Find all relatively prime solution of $x^2 + y^2 = z^2$ with $0 < z < 30$.
(7) Solve the equation $12x + 15 \equiv 0(mod\ 45)$.
(8) If $(a, p) = 1$, p is prime , then prove that $a^{p-1} \equiv 1(mod\ p)$.

Que.3 (a) Let g be a positive integer greater than 1 then prove that every positive integer a can can be written uniquely in the form $a = c_n g^n + c_{n-1} g^{n-1} + ... + c_1 g + c_0$, where $n \geq 0$, $c_i \in \mathbb{Z}$, $0 \leq c_i < g$, $c_n \neq 0$. 5

(b) Prove that $(a, b)|c| = (ac, bc)$, $\forall\ c \neq 0$. 3

OR

Que.3 (c) If P_n is n^{th} prime number then prove that $P_n < 2^{2^n}$, $\forall\, n \in \mathbb{N}$. 5

(d) Prove that $(a, b) = 1$ iff $\exists\ x, y \in \mathbb{Z}$ such that $xa + yb = 1$. 3

Que.4 (a) Prove that every prime factor of F_n $(n > 2)$ is of the form $2^{n+2}t + 1$, for some integer t. 5

(b) If $2^m - 1$ is prime then prove that m is also prime .Does the converse hold ? verify it . 3

OR

Que.4 (c) Prove that $S(a) < a\sqrt{a}$, $\forall\ a > 2$. 4

(d) Prove that $(u_m, u_n) = u_{(m,n)}$ .Hence Prove that u_m/u_niff m/n. 4

Que.5 (a) Prove that a general integer solution of $x^2 + y^2 + z^2 = w^2$, $(x, y, z, w) = 1$ is given by $x = (a^2 - b^2 + c^2 - d^2)$, $y = 2ab - 2cd$, $z = 2ad + 2bc$, $w = a^2 + b^2 + c^2 + d^2$. 5

(b) Prove that a positive integer n is divided by 9 iff the sum of its digits is divisible by 9. 3

OR

Que.5 (c) Prove that the equation $x^4 + y^4 = z^2$ has no solution with nonzero positive integers x , y , z . Hence prove that $x^4 - 4y^4 = z^2$ has no nonzero positive integer solution. 5

(d) If $(a, b) = d$,then prove that general solution of $ax + by = c$ can be written as $x = x_0 + \frac{b}{d}t$; $y = y_0 - \frac{a}{d}t$, where $t \in \mathbb{Z}$ and $x = x_0, y = y_0$ is a particular solution of $ax + by = c$ 3

Que.6 (a) Prove that m is prime iff $\phi(m) + S(m) = mT(m)$. 4

(b) State and prove Sun-Tsu theorem. 4

OR

Que.6 (c) Prove that Euler's function is multiplicative function and hence find $\phi(1708)$. 5

(d) Solve $6x + 15y \equiv 9(mod\ 18)$. 3

♠♠♠♠♠♠♠♠♠♠♠♠♠