V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2018-19

Subject : Mathematics   US06CMTH01   Max. Marks : 50

Real Analysis - III

Date: 05/03/2019   Timing: 10:00 am - 12:00 Noon

Instruction : The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices. 8

[1] In usual notations, the *Schlömilch-Röche* form of remainder in Maclaurin's theorem is

[A] $\dfrac{x^n(1-\theta)^{n-p}}{p(n-1)!}f^{(n)}(\theta x)$ [B] $\dfrac{x^n(1+\theta)^{n-p}}{p(n-1)!}f^{(n)}(\theta x)$

[C] $\dfrac{x^{(n-1)}(1-\theta)^{n-p}}{p(n-1)!}f^{(n)}(\theta x)$ [D] $\dfrac{x^{n-1}(1-\theta)^{n-p}}{p(n-1)!}f^{(n)}(\theta x)$

[2] If a function f defined on $[a,b]$ is
(i) continuous on $[a,b]$ (ii) differentiable on (a,b) and (iii) $f(a)=f(b)$
then the tangent at atleast one point on the curve $y=f(x)$ is

[A] parallel to the X-axis   [B] perpendicular to the X-axis
[C] parallel to the Y-axis   [D] does not exist

[3] A derivable function $f(x)$ has a maximum at c if while x passes through c, the derivative f'

[A] remains positive
[B] remains negative
[C] changes its sign from negative to positive
[D] changes its sign from positive to negative

[4] If f has a minima at c then there is some $\delta > 0$ such that $\forall x \in (c-\delta, c+\delta), x \neq c$

[A] $f(c) < f(x)$ [B] $f'(c) < f'(x)$ [C] $f(c) > f(x)$ [D] $f'(c) > f'(x)$

[5] If P is a partition of $[a,b]$ then

[A] $a \in P$ but $b \notin P$ [B] $a \notin P$ but $b \in P$
[C] $a \notin P$ and $b \notin P$ [D] $a \in P$ and $b \in P$

[6] For every bounded function f defined on $[a,b]$

[A] $\underline{\int_a^b} f.dx \leqslant \overline{\int_a^b} f.dx$ [B] $\underline{\int_a^b} f.dx \geqslant \overline{\int_a^b} f.dx$ [C] $\underline{\int_a^b} f.dx = \overline{\int_a^b} f.dx$ [D] none

[7] A continuous function over a closed interval $[a,b]$ is always

[A] an increasing function   [B] a decreasing function
[C] a constant function   [D] an integrable function

[8] An increasing function over a closed interval $[a, b]$

[A] is always an integrable function

[B] is always a differentiable function

[C] cannot be always an integrable function

[D] none

Q: 2. Answer any FIVE of the following. 10

[1] Explain the algebraic meaning of Rolle's theorem

[2] State Cauchy's Mean Value theorem

[3] Show that, $f(x) = x^2 - 4x - 5$ has a minimum at 2

[4] Show that $f(x) = 5x + 8$ cannot have a stationary point

[5] Write a common refinement of the partitions $P_1 = \{2, 3, 4, 6, 8, 12\}$ and $P_2 = \{2, 3, 9, 10, 12\}$ of $[2, 12]$.

[6] Can two partitions of $[a, b]$ be disjoint? Justify.

[7] Is an integrable function over $[a, b]$ necessarily continuous on $[a, b]$? Why?

[8] Is $f(x) = x$ an integrable function over $[0, 1]$? Justify.

Q: 3 [A] State and prove Lagrange's Mean Value theorem 5

[B] If a function f is continuous on $[a, b]$, derivable on (a, b) and $f'(x) > 0, \ \forall x \in (a, b)$ then prove that f is strictly incerasing function on $[a, b]$ 3

OR

Q: 3 [A] State and prove Taylor's theorem. 5

[B] Examine the validity of the hypothesis and the conclusion of Lagrange's Mean Value theorem for the function $f(x) = 2x^2 - 7x + 10$ on $[2, 5]$ 3

Q: 4 [A] If c is an interior point of the domain $[a, b]$ of a function f and is such that
(i) $f'(c) = f''(c) = f'''(c) = ... = f^{(n-1)}(c) = 0$ and
(ii) $f^{(n)}$ exists and is non-zero
then show that for n odd, $f(c)$ is not an extreme value, while for n even $f(c)$ is maximum or minimum according as $f^{(n)}$ is negative or positive. 5

[B] Prove that a conical tent of a given capacity will require the least amount of canvas when the height is $\sqrt{2}$ times the radius of the base. 3

OR

Q: 4 [A] Examine the function $\sin x + \cos x$ for extreme values 5

[B] Show that $\sin x(1 + \cos x)$ is maximum at $x = \frac{\pi}{3}$ 3

Q: 5 [A] Prove that if f_1 and f_2 are bounded and integrable functions on $[a, b]$, then their product $f_1 f_2$ is also bounded and integrable on $[a, b]$ 5

[B] Show that the square of an integrable function on $[a, b]$ is also integrable on $[a, b]$ 3

OR

Q: 5 [A] Prove that a necessary and sufficient condition for the integrability of a bounded function f is that to every $\epsilon > 0$ there corresponds $\delta > 0$ such that for every partition P of $[a, b]$ with $\mu(P) < \delta$,

$$U(P, f) - L(P, f) < \epsilon$$

5

[B] Show that x^2 is integrable on any interval $[0, k]$ 3

Q: 6 [A] If f_1, f_2 are integrable on $[a, b]$ and c_1 and c_2 any two constants, then prove that $c_1 f_1 + c_2 f_2$ is integrable and

$$\int_a^b (c_1 f_1 + c_2 f_2).dx = \int_a^b c_1 f_1.dx + \int_a^b c_2 f_2.dx$$

5

[B] Prove that a bounded function f is integrable on $[a, b]$, if the set of points of discontinuity has only a finite number of limit points. 3

OR

Q: 6 [A] State and prove the Second Mean Value theorem of Integral Calculus 5

[B] If f is a non-negative continuous function on $[a, b]$ and $\int_a^b f.dx = 0$ then prove that $f(x) = 0,\ \forall x \in [a, b]$ 3