V.P.& R.P.T.P.Science College.Vallabh Vidyanagar.
B.Sc. ( Semester - V ) Internal Test
US05CMTH04 ( Abstract Algebra - 1 )
Date. 7/10/2019 ; Monday 11.00 a.m. to 12.15 p.m. Maximum Marks: 25



Que.1 Fill in the blanks. 5

(1) In group $(Z_7^*, \cdot)$ , $\bar{6}^{-1} =$ ...............
(a) $\bar{6}$ (b) $\bar{3}$ (c) $\bar{4}$ (d) $\bar{2}$

(2) Cyclic group of order 13 has only ................. generator .
(a) 13 (b) 11 (c) 12 (d) 2

(3) Define $f : R^* \to R^*$ by $f(x) = x^2$ then Ker $f =$ ......................
(a) 0 (b) $\pm 1$ (c) 1 (d) $\{\pm 1\}$

(4) ................. is quotient group of $Z_{12}$ .
(a) $Z_2$ (b) $Z_8$ (c) $Z_{10}$ (d) $Z_9$

(5) Ker $\varepsilon =$ ......................
(a) $A_n$ (b) $e$ (c) $\pm 1$ (d) $S_n$

Que.2 (a) Prove that $(G, *)$ is a commutative group ,where G is a set of all subsets of $\mathbb{R}$ and operation $*$ defined as $A * B = (A - B) \cup (B - A) \quad \forall A, B \in G$ . 5

OR

Que.2 (b) Let H and K be finite subgroups of group G such that HK is a subgroup of G.Then prove that $O(HK) = \dfrac{O(H)O(K)}{O(H \cap K)}$. 5

Que.3 (a) Prove that any subgroup of a cyclic group is also cyclic group . Also prove that every subgroup of an infinite cyclic group is infinite cyclic group. 5

OR

Que.3 (b) Let G be a finite cyclic group of order $n$ , then prove that G has $\phi(n)$ generators . 5

Que.4 (a) State and prove Third isomorphism theorem . 5

OR

Que.4 (b) Let $G = (a)$ be a finite cyclic group of order $n$ .Then prove that the mapping $\theta : G \to G$ defined by $\theta(a) = a^m$ is an automorphism of $G$ iff $m$ is relatively prime to $n$ . 5

Que.5 (a) Let $G = H \times K$ be external direct product of H and K ,then prove that $G/K' \simeq H$ , where $K' = \{(e_H, k)/k \in K\}$ . 5

OR

Que.5 (b) Prove that G is direct product of subgroups H and K iff (i) every $x \in G$ can be uniquely expressed as $x = hk$ , $h \in H$ , $k \in K$ (ii) $hk = kh$ , $h \in H$ , $k \in K$. 5

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