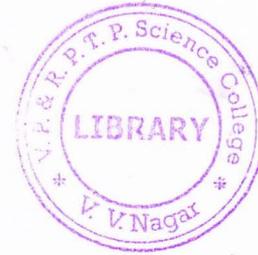


Que.1 Fill in the blanks.

5

- (1) In group  $(Z_7^*, \cdot)$ ,  $\bar{6}^{-1} = \dots\dots\dots$   
 (a)  $\bar{6}$  (b)  $\bar{3}$  (c)  $\bar{4}$  (d)  $\bar{2}$
- (2) Cyclic group of order 13 has only  $\dots\dots\dots$  generator .  
 (a) 13 (b) 11 (c) 12 (d) 2
- (3) Define  $f : R^* \rightarrow R^*$  by  $f(x) = x^2$  then  $\text{Ker } f = \dots\dots\dots$   
 (a) 0 (b)  $\pm 1$  (c) 1 (d)  $\{\pm 1\}$
- (4)  $\dots\dots\dots$  is quotient group of  $Z_{12}$  .  
 (a)  $Z_2$  (b)  $Z_8$  (c)  $Z_{10}$  (d)  $Z_9$
- (5)  $\text{Ker } \varepsilon = \dots\dots\dots$   
 (a)  $A_n$  (b)  $e$  (c)  $\pm 1$  (d)  $S_n$



Que.2 (a) Prove that  $(G, *)$  is a commutative group ,where  $G$  is a set of all subsets of  $\mathbb{R}$  and operation  $*$  defined as  $A * B = (A - B) \cup (B - A) \forall A, B \in G$  .

5

OR

Que.2 (b) Let  $H$  and  $K$  be finite subgroups of group  $G$  such that  $HK$  is a subgroup of  $G$ .Then prove that  $O(HK) = \frac{O(H)O(K)}{O(H \cap K)}$ .

5

Que.3 (a) Prove that any subgroup of a cyclic group is also cyclic group . Also prove that every subgroup of an infinite cyclic group is infinite cyclic group.

5

OR

Que.3 (b) Let  $G$  be a finite cyclic group of order  $n$  , then prove that  $G$  has  $\phi(n)$  generators .

5

Que.4 (a) State and prove Third isomorphism theorem .

5

OR

Que.4 (b) Let  $G = \langle a \rangle$  be a finite cyclic group of order  $n$  .Then prove that the mapping  $\theta : G \rightarrow G$  defined by  $\theta(a) = a^m$  is an automorphism of  $G$  iff  $m$  is relatively prime to  $n$  .

5

Que.5 (a) Let  $G = H \times K$  be external direct product of  $H$  and  $K$  ,then prove that  $G/K' \simeq H$  , where  $K' = \{(e_H, k)/k \in K\}$  .

5

OR

Que.5 (b) Prove that  $G$  is direct product of subgroups  $H$  and  $K$  iff (i) every  $x \in G$  can be uniquely expressed as  $x = hk$  ,  $h \in H$  ,  $k \in K$  (ii)  $hk = kh$  ,  $h \in H$  ,  $k \in K$  .

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